Complementation of Coalgebra Automata

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Goal of the Talk

Theorem

The class of languages recognisable by $\mathcal{T}$-coalgebra automata is closed under taking complements.

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The class of languages recognisable by $\mathcal{T}$-coalgebra automata is closed under taking complements.

- $\mathcal{T}$ preserves weak pullbacks
- $\mathcal{T}$ restricts to finite sets
- $\mathcal{T}$ is standard

Outline

1. One Step Complementation Lemma
   - Moss' Modality
   - Boolean Dual of Moss' Modality
2. Game Bisimulation
   - Parity Graph Games
   - Basic Sets and Local Games
   - Powers and Game Normalisation
   - Game Bisimulation
3. Complementation Lemma for Coalgebra Automata
   - Coalgebra Automata
   - Complementation of Trans-alternating Automata
   - Equivalence of Transalternating and Alternating Automata

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Moss' Modality

Definition (Moss' Modality)

- $\nabla\alpha \in \mathcal{L}\mathcal{T}Q$ where $\alpha \in \mathcal{T}\mathcal{L}Q$

where $\mathcal{L}$ is the functor taking a set Q to the set of bounded lattice terms $t ::= q \in Q \mid \top \mid \bot \mid t \wedge t \mid t \vee t$ over Q

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- $\nabla\alpha \in \mathcal{L}\mathcal{T}Q$ where $\alpha \in \mathcal{T}\mathcal{L}Q$
- Let $\mathbb{S} = \langle S, \sigma : S \to \mathcal{T}S, s_I \rangle$ and $s \in S$, then

$$\mathbb{S}, s \Vdash \nabla\alpha \text{ iff } (\sigma(s), \alpha) \in \overline{\mathcal{T}}(\Vdash)$$

where $\mathcal{L}$ is the functor taking a set Q to the set of bounded lattice terms $t ::= q \in Q \mid \top \mid \bot \mid t \wedge t \mid t \vee t$ over Q

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Example

For $\mathcal{T} = \mathcal{P}_\omega$, we get $\nabla\alpha \equiv \Box \bigvee \alpha \wedge \bigwedge \Diamond[\alpha]$

Positive Coalgebraic Logics

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- $\mathcal{L}Q$ is the set of *depth-zero formulas*
- $\mathcal{L}\mathcal{T}_\omega^\nabla \mathcal{L}Q$ is the set of *depth-one formulas*

where

- $\mathcal{T}_\omega(X) := \bigcup\{\mathcal{T}Y \mid Y \subseteq_\omega X\}$ is the finitary version of $\mathcal{T}$
- $\mathcal{T}_\omega^\nabla(X) := \{\nabla\alpha \mid \alpha \in \mathcal{T}_\omega X\}$

One-Step Semantics of Positive Coalgebraic Logics

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- $\sigma \Vdash_1^V \nabla\alpha$ if $(\sigma, \alpha) \in \overline{\mathcal{T}}(\Vdash_0^V)$

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Definition (Boolean Duals of Depth-One Formulas)

Depth-Zero Formulas a and b are boolean duals if
for all sets S, all valuations $V : Q \to \mathcal{P}(S)$, and all $s \in S$

$$s \not\Vdash_0^{V^c} a \text{ iff } s \Vdash_0^V b$$

where

- $V^c(q) := \mathcal{P}(S) \setminus V(q)$ is the complementary valuation

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One-Step Complementation Lemma

Definition (Boolean Dual of ∇)

Let $\nabla\alpha \in \mathcal{T}_\omega^\nabla Q$, define a set $D(\alpha) \subseteq \mathcal{T}_\omega \mathcal{P} Q$ as follows

$$D(\alpha) := \left\{ \Phi \in \mathcal{T}_\omega \mathcal{P}_\omega Base(\alpha) \mid (\alpha, \Phi) \notin (\overline{\mathcal{T}} \notin) \right\}$$

where $Base(\alpha \in \mathcal{T}_\omega Q)$ is the smallest $X \subseteq_\omega Q$ such that $\alpha \in \mathcal{T}_\omega X$

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$$\Delta\alpha := \bigvee \left\{ \nabla(\mathcal{T} \bigwedge)\Phi \mid \Phi \in D(\alpha) \right\}$$

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$$\Delta\alpha := \bigvee \left\{ \nabla(\mathcal{T} \bigwedge)\Phi \mid \Phi \in D(\alpha) \right\}$$

Example

For $\mathcal{T} = \mathcal{P}_\omega$, we get $\Delta\alpha := \nabla\emptyset \vee \bigwedge\{\nabla\{a\} \mid a \in \alpha\} \vee \nabla\{\bigwedge \alpha, \top\}$.

One-Step Complementation Lemma

Theorem (One-Step Complementation Lemma)

For all $\alpha \in \mathcal{T}_\omega Q$, $\nabla\alpha$ and $\Delta\alpha$ are Boolean duals.

For all sets S, all valuations $V : Q \to \mathcal{P}(S)$, and all $s \in S$, $s \not\Vdash_1^{V^c} \nabla\alpha$ iff $s \Vdash_1^{V} \Delta\alpha$

One-Step Dualisation

Definition (One-Step Dualisation)

$$\delta_0 : \mathcal{L}Q \to \mathcal{L}Q$$

$$\begin{aligned} \delta_0(q) &:= q \\ \delta_0(\bigwedge \phi) &:= \bigvee \delta_0[\phi] \\ \delta_0(\bigvee \phi) &:= \bigwedge \delta_0[\phi] \end{aligned}$$

$$\delta_1 : \mathcal{L}\mathcal{T}_\omega^\nabla \mathcal{L}Q \to \mathcal{L}\mathcal{T}_\omega^\nabla \mathcal{L}Q$$

$$\begin{aligned} \delta_1(\nabla \alpha) &:= \Delta(\mathcal{T}\delta_0)\alpha \\ \delta_1(\bigwedge \phi) &:= \bigvee \delta_1[\phi] \\ \delta_1(\bigvee \phi) &:= \bigwedge \delta_1[\phi] \end{aligned}$$

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Corollary

For any $a \in \mathcal{L}\mathcal{T}_\omega^\nabla \mathcal{L}Q$, the depth-one formulas a and $\delta_1(a)$ are Boolean duals.

For all sets S, all valuations $V : Q \to \mathcal{P}(S)$, and all $s \in S$, $s \not\Vdash_1^{V^c} a$ iff $s \Vdash_1^V \Delta\delta_1(a)$

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Parity Graph Games

Definition (Arena)

Arenas of parity graph games are structures

$$\mathcal{G} = \langle V_0, V_1, E, v_I, \Omega : V \to \mathbb{N} \rangle$$

- sets $V = V_0 \uplus V_1$ of positions
- an edge relation $E \subseteq V \times V$
- an initial position $v_I \in V$
- a priority function $\Omega : V \to \mathbb{N}$ with finite range

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Definition (Winning Condition)

Player $\Pi \in \{0, 1\}$ ($\Sigma = 1 - \Pi$) wins a total play p of $\mathcal{G}$ iff

- p finite and $last(p) \in V_\Pi$
- p infinite and largest priority occuring infinitely often has parity Π

Basic Sets

- $\mathcal{G} = \langle V_0, V_1, E, v_I, \Omega : V \to \mathbb{N} \rangle$

Definition

We call a set $B \subseteq V$ *basic* if

1. $v_I \in B$
2. any total play from $v \in B$ either ends in a terminal position or it passes through another position in B
3. $v \in B$ iff $\Omega(v) > 0$

Local Games

- $\mathcal{G} = \langle V_0, V_1, E, v_I, \Omega \rangle$ with basic set $B \subseteq V$, $b \in B$

Definition (Local Game Trees)

$$T^b = \langle V_0^b, V_1^b, E^b, (b) \rangle$$

- $V^b := \left\{ \begin{array}{l} \beta \in V^* \mid \mathit{first}(\beta) = b, \\ \forall i < |\beta|.\beta(i) \in B \implies i = 0 \text{ or } i = |\beta| - 1 \end{array} \right\}$
- $V_\Pi^b := \{\beta \in V^b \mid \mathit{last}(\beta) \in V_\Pi\}$ for both $\Pi \in \{0, 1\}$
- $E^b(\beta) := \{\beta v \mid v \in E(\beta)\}$

Powers

- $\mathcal{G} = \langle V_0, V_1, E, v_I, \Omega' \rangle$ with basic set $B \subseteq V$
- $b \in B,\ T^b = \langle V_0^b, V_1^b, E^b, (b) \rangle$

Definition (Powers)

We define the power $P_\Pi(b) \subseteq B$ of $\Pi \in \{0, 1\}$ ($\Sigma = 1 - \Pi$) as

- If $\beta \in \mathit{Leaves}(T^b)$, put $P_\Pi(\beta) := \Big\{ \{\mathit{last}(\beta)\} \Big\}$
- If $\beta \notin \mathit{Leaves}(T^b)$, put

$$P_\Pi(\beta) := \begin{cases} \bigcup \{ P_\Pi(\gamma) \mid \gamma \in E^b(\beta) \} & \text{if } \beta \in V_\Pi^b \\ \Big\{ \bigcup_{\gamma \in E^b(\beta)} Y_\gamma \mid Y_\gamma \in P_\Pi(\gamma),\ \text{all } \gamma \Big\} & \text{if } \beta \in V_\Sigma^b \end{cases}$$

- $P_\Pi(b) := P_\Pi(\ (b)\ )$

Powers

- $\mathcal{G} = \langle V_0, V_1, E, v_I, \Omega \rangle$, basic set $B \subseteq V$
- $\Pi \in \{0, 1\}$, $\Sigma = 1 - \Pi$

Proposition

Let W be a subset of B. Then the following are equivalent:

1. $W \in P_\Pi(b)$;
2. Π *has a surviving strategy f in $\mathcal{G}^b$ such that W is the set of next basic positions in some play consistent with f*

Powers

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Proposition

The following are equivalent

1. $\emptyset \in P_\Pi(b)$
2. $P_\Sigma(b) = \emptyset$
3. Π *has a local winning strategy in $\mathcal{G}^b$*

Game Bisimulation

- $\mathcal{G} = \langle V_0, V_1, E, \Omega \rangle$, basic set $B \subseteq V$, $\Pi \in \{0, 1\}$
- $\mathcal{G}' = \langle V_0', V_1', E', \Omega' \rangle$, basic set $B' \subseteq V'$, $\Pi' \in \{0', 1'\}$

Definition (Game Simulation)

A Π, Π'-game simulation is a relation $Z \subseteq B \times B'$ such that for all $v \in V$ and $v' \in V'$ with vZv', Z satisfies the **structural conditions**

- (pro) $\forall W \in P_{\Pi}^{\mathcal{G}}(v). \exists W' \in P_{\Pi'}^{\mathcal{G}'}(v'). \forall w' \in W'. \exists w \in W.\ wZw'$,
- (op) $\forall W' \in P_{\Sigma'}^{\mathcal{G}'}(v'). \exists W \in P_{\Sigma}^{\mathcal{G}}(v). \forall w \in W. \exists w' \in W.\ wZw'$,

and the **priority conditions**

- (parity) $\Omega(v)\ mod\ 2 = \Pi$ iff $\Omega'(v')\ mod\ 2 = \Pi'$,
- (contraction) for all $v, w \in V$ and $v', w' \in V'$ with vZv' and wZw', $\Omega(v) \leq \Omega(w)$ iff $\Omega(v') \leq \Omega(w')$.

Game Bisimulation

- $\mathcal{G} = \langle V_0, V_1, E, \Omega \rangle$, basic set $B \subseteq V$, $\Pi \in \{0, 1\}$
- $\mathcal{G}' = \langle V_0', V_1', E', \Omega' \rangle$, basic set $B' \subseteq V'$, $\Pi' \in \{0', 1'\}$

Definition (Game Bisimulation)

$Z \subseteq B \times B'$ is a Π,Π'-game bisimulation if

- Z is a Π,Π'-game simulation
- $Z^{\smile}$ is a Π',Π-game simulation

Game Bisimulation

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Theorem

If $Z \subseteq B \times B$ Π,Π'-game bisimulation between parity graph games $\mathcal{G}$ and $\mathcal{G}'$, then

$$\textit{if } vZv' \textit{ then } v \in Win_{\Pi}(\mathcal{G}) \iff v' \in Win_{\Pi'}(\mathcal{G}')$$

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Alternating Automata, Syntax

Definition (Alternating Automata in Logical Form)

Alternating $\mathcal{T}$-automata are structures

$$\mathbb{A} = \langle Q, \theta : Q \to \mathcal{L}\mathcal{T}^{\nabla} Q, q_I, \Omega \rangle$$

consisting of

- a *finite* set Q of states
- a transition function $\theta : Q \to \mathcal{L}\mathcal{T}^{\nabla} Q$
- an initial state $q_I \in Q$
- a priority function $\Omega : Q \to \mathbb{N}$

Alternating Automata, Semantics

- $\mathbb{A} = \langle Q, \theta : Q \to \mathcal{LT}^{\nabla} Q, q_I, \Omega \rangle$ is an alternating automaton
- $\mathbb{S} = \langle S, \sigma : S \to \mathcal{T} S, s_I \rangle$ is a pointed $\mathcal{T}$-coalgebra

Definition

Acceptance games are parity graph games

$$\mathcal{G}(\mathbb{A}, \mathbb{S}) = \langle V_\exists, V_\forall, E, (q_I, s_I), \Omega_{\mathcal{G}} \rangle$$

Position		Sets of Admissible Moves	$\Omega_{\mathcal{G}}$
$(q, s) \in Q \times S$	-	$\{(\theta(q), s)\}$	$\Omega(q)$
$(\bigwedge \tau, s) \in \mathcal{LT}^{\nabla} Q \times S$	$\forall$	$\{(q, s) \mid q \in \tau\}$	0
$(\bigvee \tau, s) \in \mathcal{LT}^{\nabla} Q \times S$	$\exists$	$\{(q, s) \mid q \in \tau\}$	0
$(\nabla \alpha, s) \in \mathcal{T}^{\nabla} Q \times S$	$\exists$	$\{Z \subseteq Q \times S \mid (\alpha, \sigma(s)) \in \overline{\mathcal{T}} Z\}$	0
$Z \subseteq Q \times S$	$\forall$	Z	0

Trans-alternating Automata

- Alternating automata: $\langle Q, \theta : Q \to \mathcal{L}\mathcal{T}^{\nabla} Q, q_I, \Omega \rangle$

Definition (Trans-alternating Automata)

$$\langle Q, \theta : Q \to \mathcal{L}\mathcal{T}^{\nabla}\mathcal{L} Q, q_I, \Omega \rangle$$

Definition (Acceptance Games)

similar to the acceptance games of alternating automata

Complements of Trans-alternating Automata

- $\mathbb{A} = \langle Q, \theta : Q \to \mathcal{L}\mathcal{T}^{\nabla}\mathcal{L}Q, q_I, \Omega\rangle$ is a trans-alternating aut'on

Definition (Complements of Trans-alternating Automata)

Define the complementary automaton

$$\mathbb{A}^c = \langle Q, \theta^c : Q \to \mathcal{L}\mathcal{T}^{\nabla}\mathcal{L}Q, q_I, \Omega^c\rangle$$

such that

- $\theta^c(q) := \delta_1(\theta(q))$
- $\Omega^c(q) := \Omega(q) + 1$, for all $q \in Q$.

$$\delta_0 : \mathcal{L}Q \to \mathcal{L}Q$$

$$\begin{array}{rcl} \delta_0(q) & := & q \\ \delta_0(\bigwedge \phi) & := & \bigvee \delta_0[\phi] \\ \delta_0(\bigvee \phi) & := & \bigwedge \delta_0[\phi] \end{array}$$

$$\delta_1 : \mathcal{L}\mathcal{T}_{\omega}^{\nabla}\mathcal{L}Q \to \mathcal{L}\mathcal{T}_{\omega}^{\nabla}\mathcal{L}Q$$

$$\begin{array}{rcl} \delta_1(\nabla\alpha) & := & \Delta(\mathcal{T}\delta_0)\alpha \\ \delta_1(\bigwedge \phi) & := & \bigvee \delta_1[\phi] \\ \delta_1(\bigvee \phi) & := & \bigwedge \delta_1[\phi] \end{array}$$

Complements of Trans-alternating Automata

Theorem

For every trans-alternating automaton $\mathbb{A}$, the automaton $\mathbb{A}^c$ accepts precisely those pointed $\mathcal{T}$-coalgebras that are rejected by $\mathbb{A}$.

Trans-alternating and Alternating Automata

Theorem

There is an effective translation between

1. *Alternating Automata*
2. *Trans-alternating Automata*

$1 \rightarrow 2$ *is trivial*

Trans-alternating and Alternating Automata

- Alternating automata: $\langle Q, \theta : Q \to \mathcal{L}\mathcal{T}^{\nabla} Q, q_I, \Omega \rangle$
- Trans-alternating automata: $\langle Q, \theta : Q \to \mathcal{L}\mathcal{T}^{\nabla} \mathcal{L} Q, q_I, \Omega \rangle$

Definition (Semi-Transalternating Automata)

$$\langle Q, \theta : Q \to \mathcal{L}\mathcal{T}^{\nabla} \mathcal{S} Q, q_I, \Omega \rangle$$

where

- $\mathcal{S}$ is the functor taking a set Q to the set of bounded meet-semilattice terms $t ::= q \in Q \mid \top \mid t \wedge t$ over Q

Definition (Acception Games)

similar to the acceptance games of alternating automata

Trans-alternating and Alternating Automata

Theorem

There is an effective translation between

1. *Alternating Automata*
2. *Trans-alternating Automata*
3. *Semi-Transalternating Automata*

We showed $1 \rightarrow 2 \rightarrow 3 \rightarrow 1$

Size Matters

Theorem

For every alternating automaton $\mathbb{A}$ *with* n *states* **there is a complementing alternating automaton** $\mathbb{A}^c$ **with** $2^n \times n$ **states**.

Size Matters

Theorem

For every alternating automaton $\mathbb{A}$ *with* n *states* **there is a complementing alternating automaton** $\mathbb{A}^c$ **with** $2^n \times n$ **states**.

Theorem

If $\mathcal{T}$ *is such that* $\Delta\alpha \in \mathcal{L}\mathcal{T}^{\nabla}Q$ *for any* $\nabla\alpha \in \mathcal{T}^{\nabla}Q$*, then for any alternating* $\mathcal{T}$*-automaton of* n *states* **there is a complementing alternating automaton with at most** $n + c$ **states**, *for some constant* c.

Some Conclusions

Summary

- Effective Complementation Procedure for Coalgebra Automata
- Coinductive Method of Game (Bi)Simulation for (some) Parity Graph Games

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Corollaries

- (Boolean) Coalgebraic Logic is Negation-free
- Correspondence between (Second-Order Monadic) Coalgebraic Logic and Coalgebra Automata

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Open Questions

- Categorical Nature of the Correspondence
- Characterisation of Game (Bi)Similarity

Conclusions and References

Thank You

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