Non-Strongly Stable Orders Also Define Interesting Simulation Relations

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Outline

- We present two new notions of simulation which can be defined as coalgebraic simulations.
  - Covariant-contravariant simulation: I/O Automata.
  - Conformance simulation: reducing non-determinism.
- In order to define them in a proper way we need an order with good enough properties.

- Stability was guaranted in [HughesJacobs04] by a stronger condition (“right-stability”).
  - We have shown that it induces a “natural” direction in the induced simulation order.
  - However, the symmetric “left-stability” also guarentees stability.
  - Other, more ellaborated “combinations” of right and left stable orders also do the work.

Bisimulations

- F : **Sets** $\to$ **Sets** can be lifted to $\mathrm{Rel}(F)$: **Rel** $\to$ **Rel**:

$$\begin{aligned}\mathrm{Rel}(F)(R) &= \{\langle u, v\rangle \in FX_1 \times FX_2 \mid \exists w \in F(R). \\ &\qquad F(r_1)(w) = u, F(r_2)(w) = v\}\end{aligned}$$

- A *bisimulation* for $c : X \longrightarrow FX$ and $d : Y \longrightarrow FY$ is a relation $R \subseteq X \times Y$ such that

$$\text{if } (x, y) \in R \text{ then } (c(x), d(y)) \in \mathrm{Rel}(F)(R)$$

Simulations

- *An order* $\sqsubseteq$ *on* F is given by a collection $\sqsubseteq_X \subseteq FX \times FX$ that is functorial (roughly, it must be preserved by renaming).

- A $\sqsubseteq$-*simulation* for $c : X \longrightarrow FX$ and $d : Y \longrightarrow FY$ is a relation $R \subseteq X \times Y$ such that

$$\text{if } (x, y) \in R \text{ then } (c(x), d(y)) \in \mathrm{Rel}_{\sqsubseteq}(F)(R),$$

where $\mathrm{Rel}_{\sqsubseteq}(F)(R)$ is $\sqsubseteq_Y \circ \mathrm{Rel}(F)(R) \circ \sqsubseteq_X$.

Stability

- $\sqsubseteq$ for F is *stable* if $\mathrm{Rel}_{\sqsubseteq}(F)$ commutes with substitution.
  - Given $f : X \longrightarrow Z$ and $g : Y \longrightarrow W$, $\mathrm{Rel}_{\sqsubseteq}(F)((f \times g)^{-1}(R)) = (Ff \times Fg)^{-1}(\mathrm{Rel}_{\sqsubseteq}(F)(R))$
- $\sqsubseteq$ for F is **right-stable** if $\forall f : X \longrightarrow Y$
$$(id \times Ff)^{-1} \sqsubseteq_Y \subseteq \coprod_{Ff \times id} \sqsubseteq_X.$$
- Right-stability is equivalent to
  - F stable and
  - $\mathrm{Rel}(F)(R) \circ \sqsubseteq_X \subseteq \sqsubseteq_Y \circ \mathrm{Rel}(F)(R)$.
- If F is right-stable,
$$\sqsubseteq_Y \circ \mathrm{Rel}(F)(R) \circ \sqsubseteq_X = \sqsubseteq_Y \circ \mathrm{Rel}(F)(R)$$

Plain Simulation

- Labeled transition systems (lts) are coalgebras for the functor $\mathcal{P}^A$.
- "Classical" simulations coincide with coalgebraic simulations for the order $\subseteq$ given by
$$\alpha \subseteq \beta \Leftrightarrow \forall a \in A,\ \alpha(a) \subseteq \beta(a).$$
- It is right-stable [HughesJacobs04].

Plain Simulation

- As a consequence of the right-stability $\subseteq$-simulations can be characterized as the $(\subseteq_Y \circ \mathrm{Rel}(F)(R))$-coalgebras. The use of $\subseteq_X$ at the lhs can be replaced by that of $\subseteq_Y$ at the rhs:
  - $\subseteq_X$ “adds new successors to $c(x)$”.
  - $\subseteq_Y$ “removes successors of $d(y)$”.
  - If S' simulates S, by removing the exceeding part to S' we obtain S'' “bisimilar” to “S”.

Anti-simulations

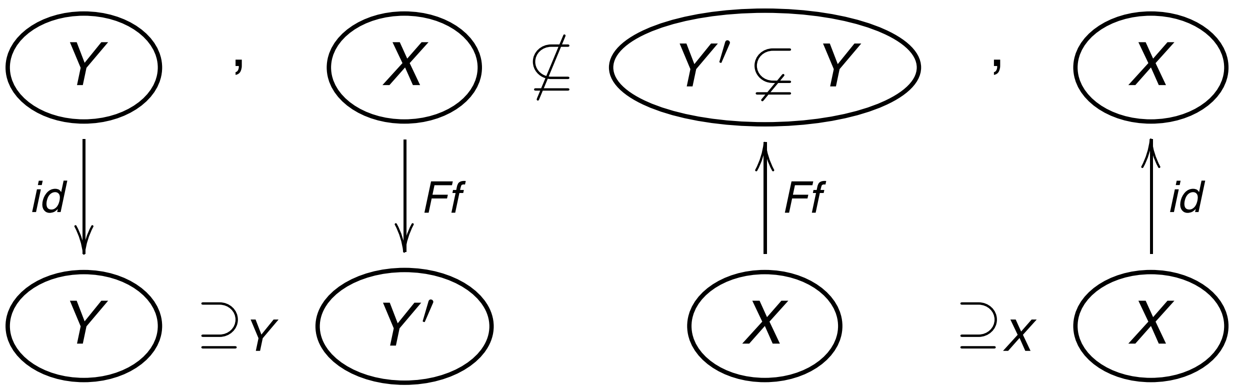
- Anti-simulations are $\supseteq$-simulations for $F = \mathcal{P}^A$ when we take

$$\alpha \supseteq \beta \Leftrightarrow \alpha(a) \supseteq \beta(a) \; \forall a \in A.$$

- If *c* “simulates” *d* then *d* “is simulated by” *c*.

- The order $\supseteq$ is not “right-stable”:

$$(id \times Ff)^{-1} \supseteq_Y \qquad \coprod_{Ff \times id} \supseteq_X$$

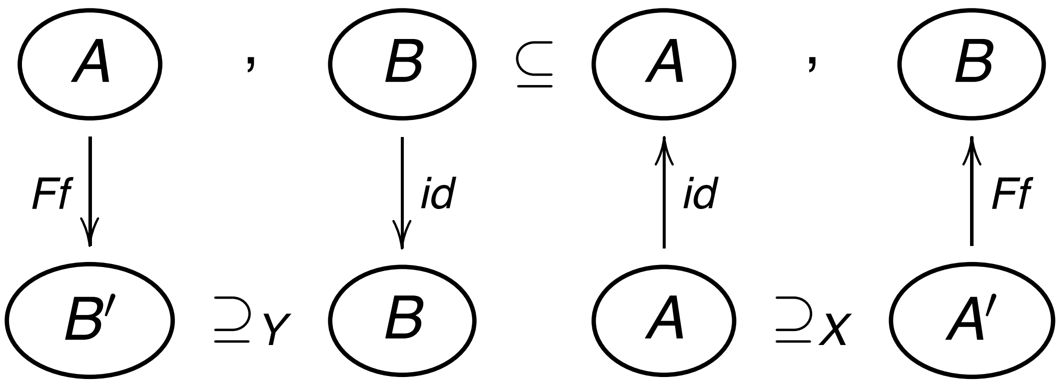


Anti-simulations

- F with $\sqsubseteq$ is **left-stable** if $\forall f : X \longrightarrow Y$,
$$(Ff \times id)^{-1} \sqsubseteq_Y \;\subseteq\; \coprod_{id \times Ff} \sqsubseteq_X.$$

- Anti-simulation is *left-stable*.

$$(Ff \times id)^{-1} \supseteq_Y \qquad\qquad \coprod_{id \times Ff} \supseteq_X$$



Relating (Trivially) Left-stable and Right-stable Orders

- F with $\sqsubseteq$ is stable iff it is stable with the inverse order $\sqsubseteq^{op}$.
- An order $\sqsubseteq$ is left-stable iff $\sqsubseteq^{op}$ is right-stable.
  - Both right-stability and left-stability give a "natural" direction to simulation relations.
- Left-stable orders have the same structural properties as right-stable ones.
  - $\subseteq$-similarity is transitive, etc.
- The composition of right (resp. left)-stable orders gives us a new right (resp. left)-stable order.

Covariant-contravariant simulations

- Related with (I/O) automata [LynchVaandrager87].
- Classic simulations are based on the definition of semantics for reactive systems.
  - All actions are considered to be inputs that the user must trigger.
- But for output actions the opposite holds.
  - Now it is the system that produces the actions that the user is forced to accept. So a bigger production imposes more constraints to the user.
  - It is natural to apply anti-simulation ideas to capture these actions.

Covariant-contravariant simulations

- Given an alphabet Act, we will consider a partition $\{Act^r, Act^l, Act^{bi}\}$ of Act.
- An (Act^r, Act^l)-**simulation** for $c : X \longrightarrow \mathcal{P}(X)^{Act}$ and $d : Y \longrightarrow \mathcal{P}(Y)^{Act}$ is a relation S such that $\forall (x, y) \in S$:
  - $\forall a \in Act^r \cup Act^{bi}$, $\forall x \xrightarrow{a} x'$ $\exists y \xrightarrow{a} y'$ with $(x', y') \in S$.
  - $\forall a \in Act^l \cup Act^{bi}$, $\forall y \xrightarrow{a} y'$ $\exists x \xrightarrow{a} x'$ with $(x', y') \in S$.
- This gives us a single framework combining plain simulations, anti-simulations and bisimulations.

Covariant-contravariant simulations

- (Act^r, Act^l)-simulations can be defined as the coalgebraic simulations for the order ${}_{Act^r}\sqsubseteq_{Act^l}$ where, for each set X and $\alpha, \alpha' : Act \longrightarrow \mathcal{P}(X)$, we have
  $\alpha \;{}_{Act^r}\sqsubseteq_{Act^l}\; \alpha' \;\;\Leftrightarrow:$
  - for all $a \in Act^r \cup Act^{bi}$, $\alpha(a) \subseteq \alpha'(a)$, and
  - for all $a \in Act^l \cup Act^{bi}$, $\alpha(a) \supseteq \alpha'(a)$.

- ${}_{Act^r}\sqsubseteq_{Act^l}$ is stable.
  - It can be "decomposed" as a product of both right-stable and left-stable orders.
  - However it is neither right-stable nor left-stable.

Conformance simulations

- They behave as plain simulations allowing the extension of the set of actions offered by a process:

  $$a < a + b$$

- But a process can also be "improved" by reducing the nondeterminism in it.

  $$ap + aq < ap$$

- A **conformance simulation** between $c : X \longrightarrow \mathcal{P}(X)^A$ and $d : Y \longrightarrow \mathcal{P}(Y)^A$, is a relation R such that $pRq \Rightarrow$
  - $\forall a \in A, p \xrightarrow{a} \Rightarrow q \xrightarrow{a}$.
  - $\forall a \in A\ (q \xrightarrow{a} q' \wedge p \xrightarrow{a}) \Rightarrow p \xrightarrow{a} p'$ and $p' R q'$.

Conformance simulations

- Conformance simulations can be defined as the coalgebraic $\sqsubseteq^{Conf}$-simulations, where for any set X and every $u, v : A \longrightarrow \mathcal{P}X$ and $a \in A$, we have $u \sqsubseteq^{Conf}_X v \iff$
  - Either $u(a) = \emptyset$, or
  - $u(a) \supseteq v(a)$ and $v(a) \neq \emptyset$.
- $\sqsubseteq^{Conf}$ is stable.
  - However it is neither right-stable nor left-stable.

Side stable orders

- We observed that in the proof of stability of the order defining covariant-contravariant simulations we work with each class of the partition of *Act* separately.

- If we have an order $\sqsubseteq$ defined over F^A we can try to split it into a family of orders $\sqsubseteq^a$ over F:
  An order $\sqsubseteq$ over a functor F^A is **action-distributive** if there exists a family of orders $\sqsubseteq^a$ on F such that
  $$f \sqsubseteq g \iff f(a) \sqsubseteq^a g(a) \text{ for all } a \in A.$$

Side stable orders

- A side stable order is just an action-distributive order such that each component is either right-stable or left-stable.
- If $\sqsubseteq = \prod_{a \in A} \sqsubseteq^a$ and is stable for all $a \in A$ $\sqsubseteq^a$, then $\sqsubseteq$ is also stable.
  - Side stable orders are stable.
- By separating the right and the left-stable components we obtain $\sqsubseteq = \sqsubseteq^{\bar{r}} \times \sqsubseteq^{\bar{l}}$.

Side stable orders

- For any side stable order $\sqsubseteq$ on F^A, we obtain the coalgebraic simulations for $\sqsubseteq$ as the $(\sqsubseteq^{\bar{r}}_Y \circ \mathrm{Rel}(F) \circ \sqsubseteq^{\bar{l}}_X)$-coalgebras.
- Coalgebraic simulations for the covariant-contravariant order ${}_{Act^r}\sqsubseteq_{Act^l}$ are side stable and can be characterized as the $(\subseteq^{\bar{r}}_Y \circ \mathrm{Rel}(F) \circ \supseteq^{\bar{l}}_X)$-coalgebras.

Composition of Right-stable and Left-stable Orders

- $\sqsubseteq^{Conf}$ can be decomposed into the composition of two orders $\prod_{a\in A}(\sqsubseteq^{a,\neg\emptyset})$ and $\prod_{a\in A}(\sqsubseteq^{a,\emptyset})$ that commute with each other and such that $\prod_{a\in A}(\sqsubseteq^{a,\emptyset})$ is right-stable and $\prod_{a\in A}(\sqsubseteq^{a,\neg\emptyset})$ is left-stable.
- We obtain this decomposition by observing that if we consider the orders:
  - $x_1 \sqsubseteq^{C\emptyset} x_2 \Leftrightarrow x_1 = \emptyset$ or $x_1 = x_2$.
  - $x_1 \sqsubseteq^{C\neg\emptyset} x_2 \Leftrightarrow x_1 \supseteq x_2$ and $x_2 \neq \emptyset$, or $x_1 = x_2$.
  - $x_1 \sqsubseteq^{C} x_2 \Leftrightarrow x_1 \sqsubseteq^{C\neg\emptyset} x_2$ or $x_1 \sqsubseteq^{C\emptyset} x_2$.

  We have that $\sqsubseteq^{C\emptyset}$ and $\sqsubseteq^{C\neg\emptyset}$ commute with each other and

  $$\sqsubseteq^{C*} = \sqsubseteq^{C\emptyset} \circ \sqsubseteq^{C\neg\emptyset} = \sqsubseteq^{C\neg\emptyset} \circ \sqsubseteq^{C\emptyset}$$

Composition of Right-stable and Left-stable Orders

- Given $\sqsubseteq^r$ that is right-stable on F and $\sqsubseteq^l$ that is left-stable, and commute with each other, their composition defines a stable order on F.
- Moreover, the coalgebraic simulations for $\sqsubseteq = \sqsubseteq^r \circ \sqsubseteq^l$ can be characterized as the $(\sqsubseteq^r \circ \mathrm{Rel}(F)(R) \circ \sqsubseteq^l)$-coalgebras.
- Coalgebraic simulations for the conformance order $\sqsubseteq^{Conf}$ are stable and can be characterized as the $(\prod_{a \in A}(\sqsubseteq_Y^{a,\neg\emptyset}) \circ \mathrm{Rel}(F)(R) \circ \prod_{a \in A}(\sqsubseteq_X^{a,\emptyset}))$-coalgebras.

- We have presented two new simulation orders.
  - Both of them can be factorized into the composition of a right-stable and a left-stable component, and so are proved to be stable.
- Right-stability is an assymetric property.
  - We can use it to direct the simulation orders in a “natural” way.
  - But dualizing the right-stability condition we obtain left-stability with the same good properties.
  - By combining both right-stable and left-stable orders in several ways we can still preserve stability.

Future work

- We plan to continue the study of the two new simulation notions once we have characterized them as good coalgebraic simulations.
  - We are interested in integrating them into our unified presentation of the semantics for processes.
- We will continue with the study of stability looking for a complete algebra of combinations of side stable orders.