Coalgebra of market games

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Outline

Outline

A process is a coalgebra

$$X \xrightarrow{R} A \Rightarrow M(B \times X)$$

where

- A — controls
- B — measurements
- X — states

A process is a coalgebra

$$A \times X \xrightarrow{R} M(B \times X)$$

where

- A — controls
- B — measurements
- X — states

A process is a coalgebra

$$A \times X \stackrel{R}{\longrightarrow\!\!\!\!\!\!\!+} B \times X$$

where

- A — controls
- B — measurements
- X — states

Feedback and stable control

$$\frac{A \times X \xrightarrow{R} B \times X \qquad\qquad B \times X \xrightarrow{\phi} A}{\dfrac{A \times X \xrightarrow{R} B \times X \xrightarrow{\phi} A}{\begin{array}{ccc} X & \xrightarrow{\gamma = Fix_A(\phi \circ R)} & A \\ {\scriptstyle \langle \gamma, \mathrm{id} \rangle} \downarrow & & \uparrow {\scriptstyle \phi} \\ A \times X & \xrightarrow[R]{} & B \times X \end{array}}}$$

A game is a coalgebra

$$A \times X \overset{\varrho}{\longrightarrow\!\!\!\!\!\!\!\!\!\!+\;\;\;} B \times X$$

where

- $A = \prod_{i \in n} A_i$ — moves
- $B = \prod_{i \in n} B_i$ — values
- $X = \prod_{i \in n} X_i$ — positions
- $n = \{0, 1, \ldots, n-1\}$ — players

A game is a coalgebra

$$A \times X \xrightarrow{\varrho} B \times X$$

where

- $A = \prod_{i \in n} A_i$ — moves
- $B = \prod_{i \in n} B_i$ — values (ordered ring)
- $X = \prod_{i \in n} X_i$ — positions
- $n = \{0, 1, \ldots, n-1\}$ — players

Response strategy and equilibrium

$$\frac{\dfrac{\dfrac{A \times X \overset{\varrho}{\longrightarrow\!\!\!\!\!\!\!+\;\;} B \times X}{A_{-i} \times X_i \xrightarrow{RS_i} A_i}}{A \times X \xrightarrow{RS = \langle RS_i \circ \pi_i \rangle_{i \in m}} A}}{\begin{array}{c} X \xrightarrow{RS^\bullet = Fix_A(RS)} A \\ \langle RS^\bullet, \mathrm{id} \rangle \searrow \quad \nearrow RS \\ A \times X \end{array}}$$

where

$$A_{-i} = \prod_{\substack{k \in n \\ k \neq i}} A_k$$

Tasks for coalgebra of games

- analyze positions as states in X
  - coalgebra homomorphisms between games
  - position bisimilarity

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- construct equilibria as fixed points in A
  - static $1 \xrightarrow{RS^\bullet} A$ or position-wise $X \xrightarrow{RS^\bullet} A$
  - equilibrium at a stationary position $1 \xrightarrow{\langle RS^\bullet, x\rangle} A \times X$

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Outline

Standard notions

(Static: $X = 1$)

A1 - Best response strategy $RS = BR$

$$s_{-i} \; BR_i \; s_i \quad \iff \quad \forall t_i \in A_i.\; \varrho_i(t_i, s_{-i}) \le \varrho_i(s_i, s_{-i})$$

Standard notions

(Static: $X = 1$)

A1 - Best response relation

$$s \; BR \; t \iff \forall i \in n.\; s_{-i} \; BR_i \; t_i$$

Standard notions

(Static: $X = 1$)

A2 - Nash equilibrium

$$
\begin{aligned}
BR^{\bullet}s &\iff s\ BR\ s \\
&\iff \forall i \in n.\ s_{-i}\ BR_i\ s_i
\end{aligned}
$$

Standard notions

(Static: $X = 1$)

A3 - Rationalizable (undominated) profile

$$BR^*s \iff \exists t.\, BR^*t \wedge tBRs$$

Standard notions

Upshot

Nash equilibrium is

- a joint result of individual optimizations
- a social solution of a distributed problem
- noone can improve their gain on their own
- it leads beyond the "zero sum" view of the world

Standard notions

Issues

- existence of equilibrium
  - Q: Is $BR^\bullet$ empty?
- equilibrium selection
  - Q: What if $s, t \in BR^\bullet$, and i plays s_i and j plays t_j?
- social benefit of equilibrium
  - Q: Is $\sum_i \varrho^i_B(s)$ a global maximum (Pareto optimal)?

Standard notions

Issues

- existence of equilibrium
  - A: Kakutani theorem
- equilibrium selection
  - A: attractor dynamics
- social benefit of equilibrium
  - A: **program the notions of response**

Example: Bird Politics

(Chicken, Missile Crisis, Prisoners' Dilemma. . .)

For

- $i \in 2 = \{0, 1\}$
- $A_i = \{retreat, attack\}$
- $B_i = \mathbb{R}$

the payoff $A_0 \times A_1 \stackrel{\varrho}{\longrightarrow} B_0 \times B_1$ is given by

	retreat	attack
retreat	$\frac{w}{2}$ (top right); $\frac{w}{2}$ (bottom left)	0 (top right); w (bottom left)
attack	w (top right); 0 (bottom left)	$\frac{w}{2} - c$ (top right); $\frac{w}{2} - c$ (bottom left)

Example: Bird Politics

(Chicken, Missile Crisis, Prisoners' Dilemma. . .)

If $c < \frac{w}{2}$, the only Nash equilibrium $1 \stackrel{s}{\longrightarrow} A_0 \times A_1$ is

$$s = \langle \text{attack}, \text{attack} \rangle$$

with the social gain of

$$\sum \varrho(s) = w - 2c$$

Example: Bird Politics

(Chicken, Missile Crisis, Prisoners' Dilemma. . .)

The *unstable* profile

$$r \quad = \quad \langle \text{retreat}, \text{retreat} \rangle$$

would give the social gain of

$$\sum \varrho(s) \quad = \quad w$$

Static strategies
($X = 1$)

B1 - Stable strategy $RS = SR$

$$
\begin{aligned}
s_{-i}\ SR_i\ s_i \quad \iff \quad & \forall t \in A\ \forall \varepsilon > 0. \\
& (1-\varepsilon)\varrho_i(t_i, s_{-i}) + \varepsilon\varrho_i(t_i, t_{-i}) \le \\
& (1-\varepsilon)\varrho_i(s_i, s_{-i}) + \varepsilon\varrho_i(s_i, t_{-i})
\end{aligned}
$$

Static strategies
($X = 1$)

B1 - Stable strategy $RS = SR$

$$s_{-i}\ SR_i\ s_i \iff \forall t_i \in A_i.\ \varrho_i(t_i, s_{-i}) \leq \varrho_i(s_i, s_{-i}) \wedge \\ (\varrho_i(t_i, s_{-i}) = \varrho_i(s_i, s_{-i}) \Rightarrow \\ \forall t_{-i} \in A_{-i}.\ \varrho_i(t_i, t_{-i}) \leq \varrho_i(s_i, t_{-i}))$$

Static strategies

$(X = 1)$

B1 - Stable response relation

$$s \; SR \; t \quad \Longleftrightarrow \quad \forall i \in n. \; s_{-i} \; SR_i \; t_i$$

Static strategies
($X = 1$)

B2 - Stable equilibrium

$$
\begin{aligned}
SR^{\bullet}s &\iff \forall i \in n.\ s_{-i}\ SR_i\ s_i \\
&\iff \forall t \in A. \forall i \in n.\ \varrho_i(t_i, s_{-i}) \leq \varrho_i(s_i, s_{-i}) \wedge \\
&\qquad \varrho_i(s_i, t_{-i}) = \varrho_i(s_i, s_{-i}) \Rightarrow \\
&\qquad \forall t_{-i} \in A_{-i}.\ \varrho_i(t_i, t_{-i}) \leq \varrho_i(s_i, t_{-i})
\end{aligned}
$$

Static strategies

$(X = 1)$

B3 - Stable (admissible) profile

$$SR^*s \iff \exists t. SR^*t \wedge tSRs$$

Example: Bird Politics

(Chicken, Missile Crisis, Prisoners' Dilemma. . .)

If $c < \frac{w}{2}$, the only stable profile $1 \overset{s}{\rightarrowtail} A_0 \times A_1$ is

$$s \quad = \quad \langle \text{attack}, \text{attack} \rangle$$

with the social gain of

$$\sum \varrho(s) \quad = \quad w - 2c$$

Static strategies

$(X = 1)$

C1 - Uniform strategy $RS = UR$

$$s_{-i}\ UR_i\ s_i \iff s_{-i}\ BR_i\ s_i \wedge \forall t_{-i} \in A_{-i}.\ s_i\ BR_{-i}\ t_{-i} \Rightarrow t_{-i}\ BR_i\ s_i$$

Static strategies

$(X = 1)$

C2 - Uniform equilibrium

$$
\begin{aligned}
UR^{\bullet}\, s &\iff \forall i \in n.\ s_{-i}\ UR_i\ s_i \\
&\iff BR^{\bullet}\, s\ \wedge \\
&\qquad \forall i \in n.\ \forall t_{-i} \in A_{-i}.\ s_i\ BR_{-i}\ t_{-i} \Rightarrow t_{-i}\ BR_i\ s_i
\end{aligned}
$$

Static strategies
$(X = 1)$

C3 - Uniform profile

$$UR^* s \iff \exists t. UR^* t \wedge tURs$$

Example: Bird Politics

(Chicken, Missile Crisis, Prisoners' Dilemma. . .)

If $c < \frac{w}{2}$, there is no uniform equilibrium for Bird Politics (neither pure nor mixed).

Example: Bird Politics

(Chicken, Missile Crisis, Prisoners' Dilemma...)

If $c < \frac{w}{2}$, there is no uniform equilibrium for Bird Politics (neither pure nor mixed).

The Kakutani theorem does not apply because the uniform response relation is not convex.

Static strategies

$(X = 1)$

D1 - Constructive strategy $RS = CR$

$$\begin{aligned} s_{-i}\ CR_i\ s_i \quad \iff \quad & \forall t_i \in A_i.\ \varrho_i(s_i, s_{-i}) < \varrho_i(t_i, s_{-i}) \Rightarrow \\ & \exists t_{-i} \in A_{-i}.\ \varrho_{-i}(t_i, t_{-i}) > \varrho_{-i}(t_i, s_{-i}) \wedge \\ & \qquad\qquad \varrho_i(t_i, t_{-i}) < \varrho_i(s_i, s_{-i}) \end{aligned}$$

Static strategies
$(X = 1)$

D2 - Constructive equilibrium

$$CR^{\bullet}\, s \iff \forall i. s_{-i} CR_i s_i$$

Static strategies

$(X = 1)$

D3 - Uniform profile

$$CR^* s \iff \exists t. CR^* t \wedge tCRs$$

Example: Bird Politics

(Chicken, Missile Crisis, Prisoners' Dilemma. . .)

There is a constructive equilibrium, consisting of the mixed strategies favoring the Pareto optimal solution $\langle \mathit{retreat}, \mathit{retreat} \rangle$.

Outline

Coalgebra of games

Equilibrium programming

Position analysis

- Problem of coordination
- Problem of competition

Conclusion

Tasks for coalgebra of games

- analyze positions as states in X
  - coalgebra homomorphisms between games
  - position bisimilarity
- construct equilibria as fixed points in A
  - static $1 \overset{RS^\bullet}{\longrightarrow} A$ or position-wise $X \overset{RS^\bullet}{\longrightarrow} A$
  - equilibrium at a stationary position $1 \overset{\langle RS^\bullet, x\rangle}{\longrightarrow} A \times X$

Types-as-positions: Bird Politics

(Chicken, Missile Crisis, Prisoners' Dilemma...)

For

- $i \in 2 = \{0, 1\}$
- $A_i = \{retreat, attack\}$
- $B_i = \mathbb{R}$
- $X_i = [0, 1]$
  - $x_0 = \mathrm{Prob}(a_1 = retreat)$
  - $x_1 = \mathrm{Prob}(a_0 = retreat)$

the payoff $A_i \times X_i \xrightarrow{\varrho_B^i} B_i$ is given by

$$\varrho_B^i(\text{retreat}, x_i) = x_i \frac{w}{2}$$

$$\varrho_B^i(\text{attack}, x_i) = x_i \frac{w + 2c}{2} + \frac{w - 2c}{2}$$

Types-as-positions: Bird Politics

(Chicken, Missile Crisis, Prisoners' Dilemma. . .)

[. . .]

Example: Majority game

(static: $X = 1$)

For

- $i \in 2m+1 = \{0, 1, \ldots 2m\}$ — players
- $A_i = \{◀, ▶\}$ — moves
- $B_i = \{0, 1\}$ — values

the payoff $A \overset{\varrho}{\longrightarrow} B$ is

$$\varrho^i(s) = \begin{cases} 1 & \text{if } \#\{j \mid s_j = s_i\} > m \\ 0 & \text{otherwise} \end{cases}$$

Example: Majority game
(static: $X = 1$)

The equilibrium $1 \overset{s}{\to} A$ consists of the mixed strategies

$$s_i = \frac{1}{2}\blacktriangleleft + \frac{1}{2}\blacktriangleright$$

with the expected social gain of

$$\mathbb{E}\left(\sum \varrho(s)\right) = 2\sum_{k=m+1}^{2m+1} \frac{\binom{2m+1}{k}}{2^{2m+1}}k = m + \frac{1}{2}$$

Positions for coordination: Majority game

(with 1-step memory)

For

- $i \in 2m+1 = \{0, 1, \ldots 2m\}$ — players
- $A_i = \{◀, ▶\}$ — moves
- $B_i = \{0, 1\}$ — values
- $X_i = \{◀, ▶\}$ — positions
  - $\xi \in X = \prod X_i$ — initialized randomly

the game $A \times X \overset{\varrho}{\longrightarrow} B \times X$ becomes

$$\varrho_B^i(s, x) = \begin{cases} 1 & \text{if } \#\{j \mid s_j = s_i\} > m \\ 0 & \text{otherwise} \end{cases}$$

$$\varrho_X^i(s, x) = \diamond \quad \text{such that } \#\{j \mid s_j = \diamond\} > m$$

Positions for coordination: Majority game

The equilibria $X \overset{s}{\nrightarrow} A$ are the coordination policies

$$
\begin{aligned}
s_i(x) &= x \\
s'_i(x) &= \neg x
\end{aligned}
$$

which assure the social gain of

$$\sum \varrho(s) = 2m + 1$$

Example: Minority game
(static: $X = 1$)

For

- $i \in 2m+1 = \{0, 1, \ldots 2m\}$ — players
- $A_i = \{◀, ▶\}$ — moves
- $B_i = \{0, 1\}$ — values

the payoff $A \overset{\varrho}{\longrightarrow} B$ is

$$\varrho^i(s) \quad = \quad \begin{cases} 1 & \text{if } \#\{j \mid s_j = s_i\} \leq m \\ 0 & \text{otherwise} \end{cases}$$

Example: Minority game
(static: $X = 1$)

The static equilibrium $1 \overset{s}{\mathrel{\mkern3mu\vcenter{\hbox{$\scriptscriptstyle+$}}\mkern-12mu{\longrightarrow}}} A$ is again

$$s_i = \frac{1}{2}\blacktriangleleft + \frac{1}{2}\blacktriangleright$$

with the expected social gain of

$$\begin{aligned}\mathbb{E}\left(\sum \varrho(s)\right) &= \sum_{k=1}^{m} \frac{\binom{2m+1}{k}}{2^{2m+1}} k \\ &= \frac{m}{2}\end{aligned}$$

Example: Minority game

(static: $X = 1$)

- Even with the maximal gain of $\sum \varrho(s) = m$, there are $m + 1$ players with $\varrho^i(s) = 0$.

Example: Minority game

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Example: Minority game

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- Even with the maximal gain of $\sum \varrho(s) = m$, there are $m + 1$ players with $\varrho^i(s) = 0$.
- There is always a majority with an incentive to disturb the current state.
- This leads **from equilibrium to evolution**.

Positions for stabilization: Minority game

(with ℓ-step memory and d ideas)

Suppose that the player i sees the positions from

$$X_i \quad = \quad M \times S^{1+d} \times \ell$$

where

- $M = S = \{\blacktriangleleft, \blacktriangleright\}^{\ell}$ — memory, strategies, ideas
- $d = \{0, 1, \ldots, d-1\}$ — number of ideas
- $\ell = \{0, 1, \ldots, \ell-1\}$ — length of the memory

Positions for stabilization: Minority game

(with ℓ-step memory and d ideas)

A position

$$x_i = \langle \mu, \sigma^{i0}, \sigma^{i1}, \ldots, \sigma^{id}, k \rangle \in M \times S^{1+d} \times \ell = X_i$$

records

- μ — the recent ℓ minority (winning) choices
- σ^{i0} — i's current strategy (ℓ-tuple of choices)
- $\sigma^{i1}, \sigma^{i2}, \ldots, \sigma^{id}$ — i's bag of ideas for strategies
- k — the current moment in the ℓ-cycle history

Positions for stabilization: Minority game

For

- $i \in 2m+1 = \{0, 1, \ldots 2m\}$ — players
- $A_i = \{◀, ▶\}$ — moves
- $B_i = \{0, 1\}$ — values
- $X_i = M \times S^{1+d} \times \ell$ — positions
  - $\xi^i \in X_i$ — initialized randomly

the payoff $A \times X \overset{\varrho_B}{\longrightarrow\!\!\!\!\!\!\!+\;\;} B$ remains

$$\varrho_B^i(s, x) = \begin{cases} 1 & \text{if } \#\{j \mid s_j = s_i\} \leq m \\ 0 & \text{otherwise} \end{cases}$$

Positions for stabilization: Minority game

...while the position update $A \times X_i \xrightarrow{\varrho^i_X} X_i$ maps

$$\varrho^i_X(s, \langle \mu, \sigma^{i*}, k \rangle) \quad = \quad \langle \tilde{\mu}, \tilde{\sigma}^{i*}, \tilde{k} \rangle$$

so that

- $\tilde{k} = k + 1 \mod \ell$
- $\tilde{\mu} = \langle \diamond, \mu_0, \mu_1, \ldots, \mu_{\ell-2} \rangle$
  - where $\diamond$ is the minority choice, i.e. $\#\{j \mid s_j = \diamond\} \leq m$
- $\tilde{\sigma}^{i*}$ is obtained by reordering
  - $\hat{\sigma}^{i*} = \langle \sigma^{i*}_{\ell-1}, \sigma^{i*}_0, \sigma^{i*}_1, \ldots, \sigma^{i*}_{\ell-2} \rangle$
  - to maintain the invariant

$$\Delta(\tilde{\sigma}^{i0}, \tilde{\mu}) \leq \Delta(\tilde{\sigma}^{i1}, \tilde{\mu}) \leq \cdots \leq \Delta(\tilde{\sigma}^{id}, \tilde{\mu})$$

Positions for stabilization: Minority game

...while the position update $A \times X_i \xrightarrow{\varrho^i_X} X_i$ maps

$$\varrho^i_X(s, \langle \mu, \sigma^{i*}, k \rangle) = \langle \tilde{\mu}, \tilde{\sigma}^{i*}, \tilde{k} \rangle$$

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  - to maintain the invariant

$$\Delta(\tilde{\sigma}^{i0}, \tilde{\mu}) \le \Delta(\tilde{\sigma}^{i1}, \tilde{\mu}) \le \cdots \le \Delta(\tilde{\sigma}^{id}, \tilde{\mu})$$

— thus $\tilde{\sigma}^{i0}$ is the best and $\tilde{\sigma}^{id}$ the worst strategy w.r.t. $\tilde{\mu}$

Positions for stabilization: Minority game

Let the profile $X \overset{s}{\twoheadrightarrow} A$ be defined by

$$s_i(\mu, \sigma^{i*}, k) = \sigma_k^{i0}$$

i.e., each player plays his currently best strategy.

Positions for stabilization: Minority game

Evolution: refine $A \times X_i \xrightarrow{\varrho^i_X} X_i$

- Each player *randomly mutates* her state by
  - dropping her worst idea $\sigma^{i(\ell-1)} \in \{◀, ▶\}^\ell$
  - adding a random idea $\sigma' \in \{◀, ▶\}^\ell$.

at chosen intervals, or triggered by bad scores.

Positions for stabilization: Minority game

Evolution: refine $A \times X_i \xrightarrow{\varrho^i_X} X_i$

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  at chosen intervals, or triggered by bad scores.

- This leads to jointly stable populations of players.

Positions for stabilization: Minority game

Stable positions

$$\frac{\dfrac{X \xrightarrow{\langle s,\mathrm{id}\rangle} A\times X \qquad A\times X \xrightarrow{\varrho_X} X}{X \xrightarrow{\langle s,\mathrm{id}\rangle} A\times X \xrightarrow{\varrho_X} X}}{\begin{array}{ccc} 1 & \xrightarrow{\eta = Fix_X(\varrho_X \circ s)} & X \\ {\scriptstyle\eta}\downarrow & & \uparrow{\scriptstyle\varrho_X} \\ X & \xrightarrow[\langle s,\mathrm{id}\rangle]{} & A\times X \end{array}}$$

yield improved social gains $\mathbb{E}(\Sigma\varrho) > \frac{m}{2} + q$.

Example: Market game

(static: $X = 1$)

For

- $i \in n = \{0, 1, \ldots n-1\}$ — players (sellers, producers)
- $A_i = B_i = \mathbb{R}$ — moves, values

the payoff $A \overset{\varrho}{\longrightarrow} B$ is

$$\varrho^i(s) = \begin{cases} s_i - c_i & \text{if } \forall j_{\in n\setminus\{i\}}.\ s_i < s_j \\ 0 & \text{otherwise} \end{cases}$$

Example: Market game
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- $i \in n = \{0, 1, \ldots n-1\}$ — players (sellers, producers)
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the payoff $A \overset{\varrho}{\nrightarrow} B$ is

$$\varrho^i(s) \quad = \quad \begin{cases} s_i - c_i & \text{if } \forall j_{\in n \setminus \{i\}}.\ s_i < s_j \\ 0 & \text{otherwise} \end{cases}$$

where

- s_i is the market price offered by the producer i,
- c_i is the production cost of i

Example: Market game

(static: $X = 1$)

The equilibria $1 \overset{s}{\longrightarrow\!\!\!\!\!+} A$ consist of the strategies

$$s_i = c_i + \varepsilon_i$$

where $\varepsilon_i \in [p_i, q_i]$ is the desired profit.

Example: Market game

(with memory and tactics)

Marketing tactics (equilibrium selection)

- to win, find ε such that $c_i + \varepsilon < c_j + \varepsilon_j$ for all $j \neq i$

Example: Market game

(with memory and tactics)

Marketing tactics (equilibrium selection)

- to win, find ε such that $c_i + \varepsilon < c_j + \varepsilon_j$ for all $j \neq i$
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Example: Market game

(with memory and tactics)

Marketing tactics (equilibrium selection)

- to win, find ε such that $c_i + \varepsilon < c_j + \varepsilon_j$ for all $j \neq i$
  - to profit, maximize among such ε
- change the game:
  - sway the buyer to pay more than the lowest price
    - lock in, bundling, price discrimination...
  - manipulate the market information
    - advertising, branding...

Stable solution: Second price market game
(Static: $X = 1$)

For

- $i \in n = \{0, 1, \ldots n-1\}$ — players
- $A_i = B_i = \mathbb{R}$ — moves, values

the payoff $A \xrightarrow{\varrho} B$ is

$$\varrho^i(s) = \begin{cases} \lceil s_i \rceil^s - s_i & \text{if } \forall j_{\in n \setminus \{i\}}.\ s_i < s_j \\ 0 & \text{otherwise} \end{cases}$$

Stable solution: Second price market game

(Static: $X = 1$)

For

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where

$$\lceil a \rceil^\beta = \bigwedge \{b \in \beta \mid a < b\}$$

Stable solution: Second price market game

(Static: $X = 1$)

The unique equilibrium $1 \overset{s}{\nrightarrow} A$ consists of the strategies

$$s_i = c_i$$

Stable solution: Second price market game

(Static, stable, unimplementable)

The unique equilibrium $1 \overset{s}{\longrightarrow\!\!\!\!\!\!\!+} A$ consists of the strategies

$$s_i = c_i$$

i.e.,

- each player announces her production cost
- the lowest cost wins the market
- the profit is $\lceil c_i \rceil^c - c_i$
  - the second lowest cost – the lowest cost

Outline

Coalgebra of games

Equilibrium programming

Position analysis

Conclusion

Conclusion: Tasks for coalgebra of games

- analyze positions as states in X
  - coalgebra homomorphisms between games
  - position bisimilarity
- construct equilibria as fixed points in A
  - static $1 \overset{s}{\nrightarrow} A$ or position-wise $X \overset{s}{\nrightarrow} A$
  - equilibrium at a stationary position $1 \overset{\langle s,x \rangle}{\nrightarrow} A \times X$