Unfolding Grammars in Adhesive Categories

Paolo Baldan, Andrea Corradini, **Tobias Heindel**,
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A short history of unfoldings

Petri Nets	Nielsen, Plotkin, Winskel
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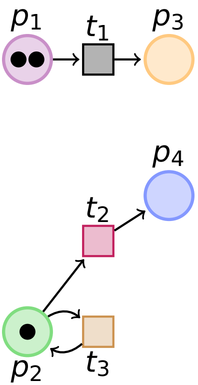
Previous work

Processes for adhesive rewriting systems
Baldan, Corradini, Heindel, König, Sobociński

FOSSACS'06

Unfolding as *“complete net behaviour”*



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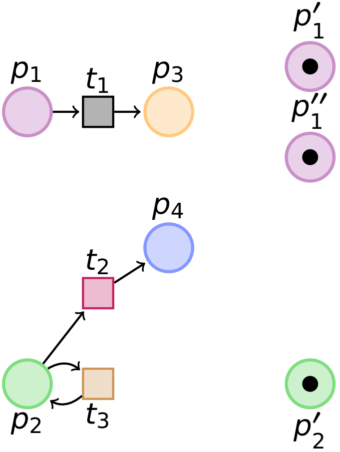

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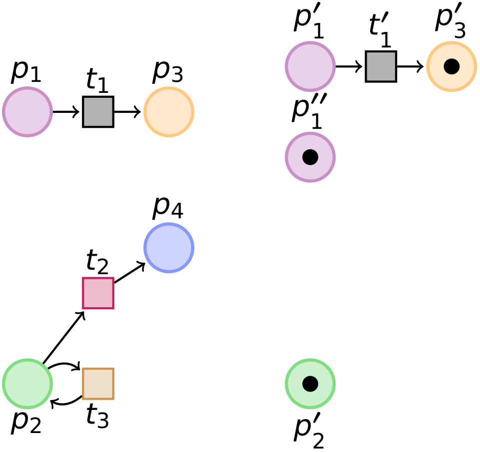

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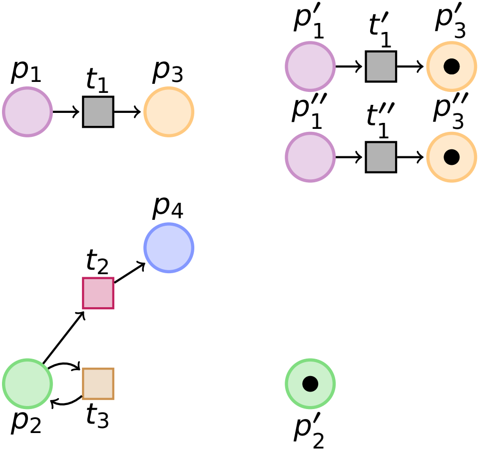

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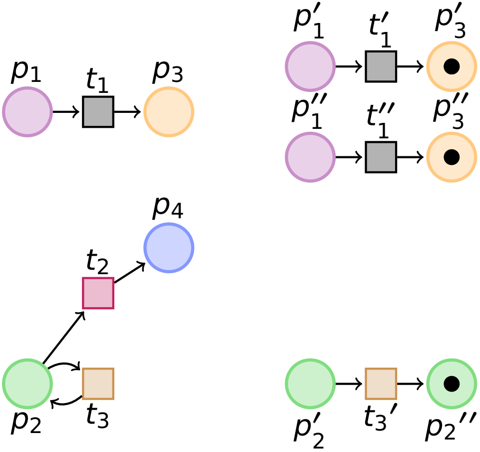

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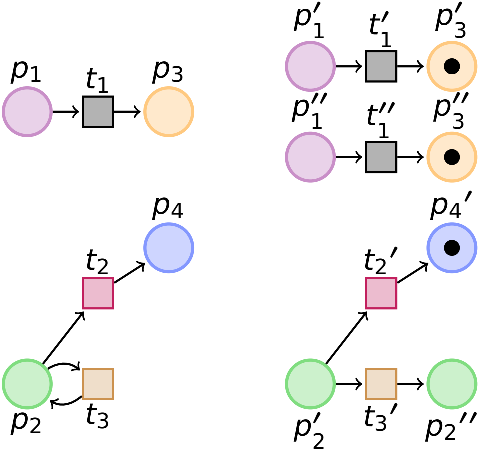

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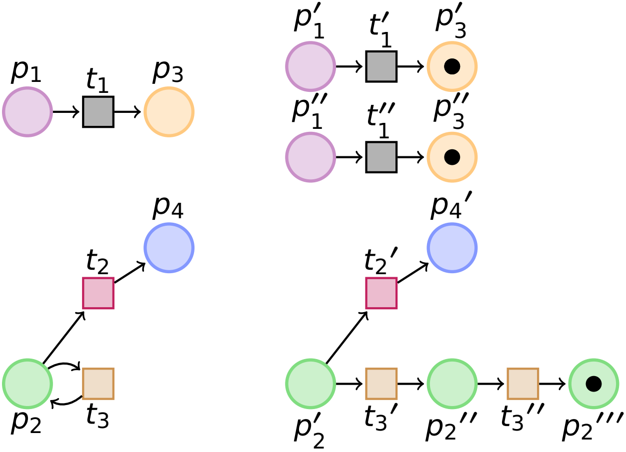

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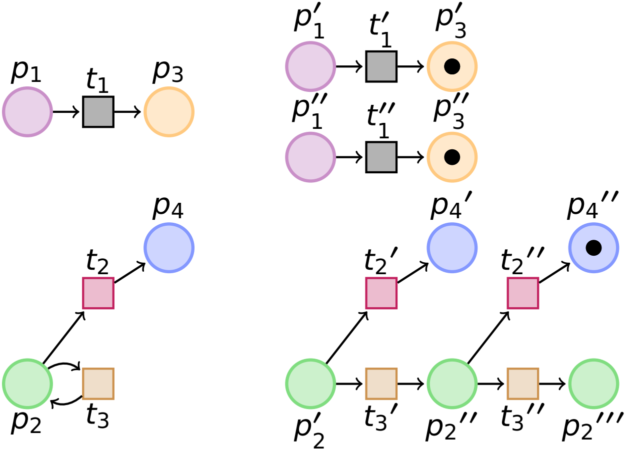

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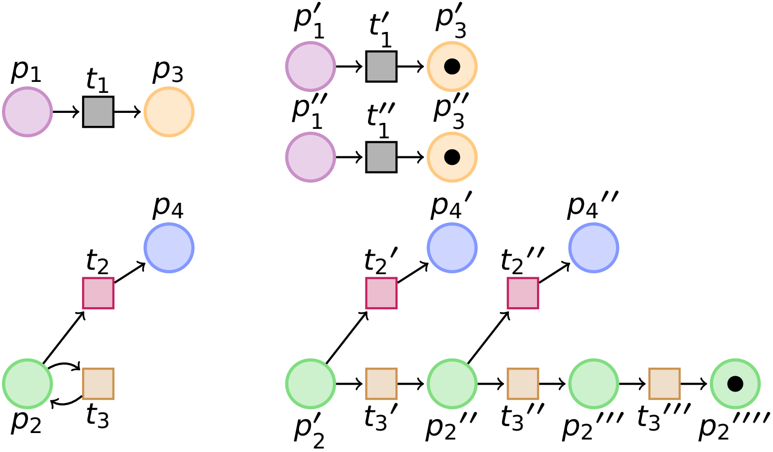

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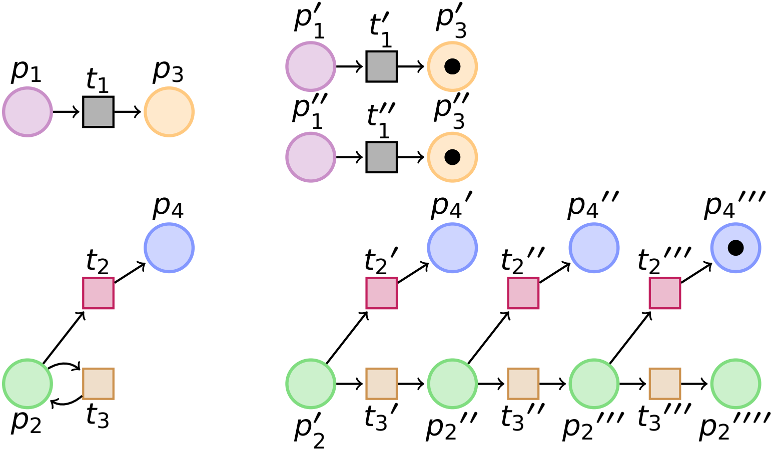

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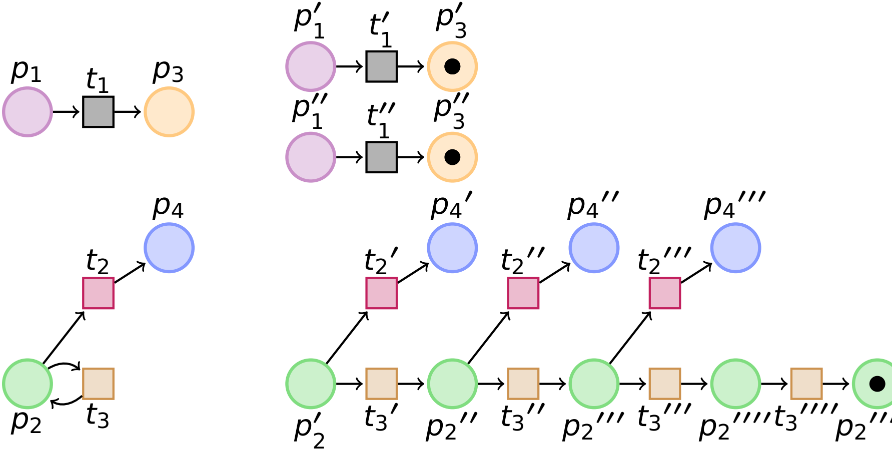

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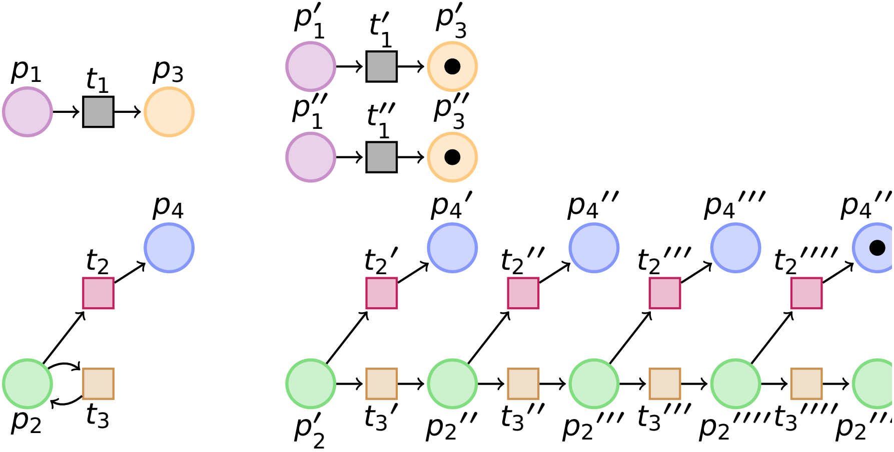

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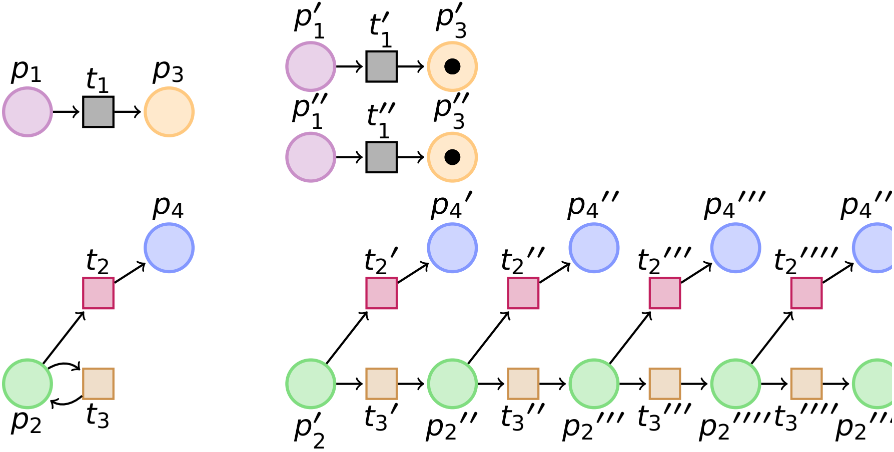

Goal: Unfolding as a coreflector

- N any net

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- $\mathcal{U}(N)$ the unfolded net

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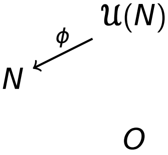

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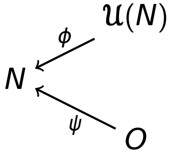

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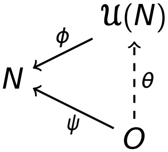

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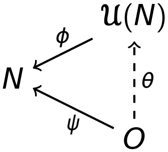

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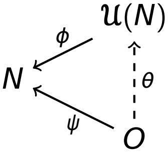

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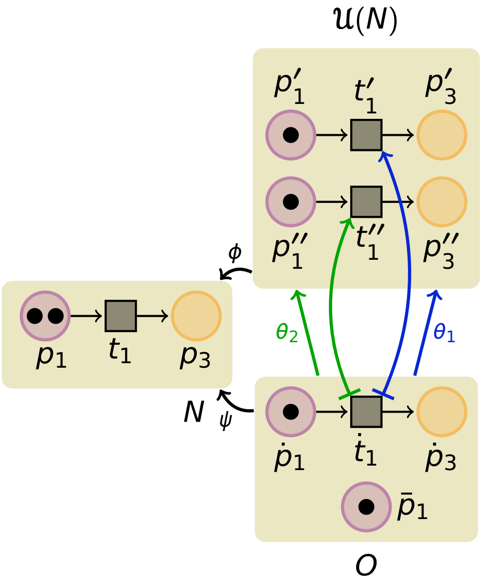
Motivation: Coreflection for “distributed” unfolding ($\rightsquigarrow$ $\mathfrak{U}$ preserves limits)

Unfolding – a coreflector or not?



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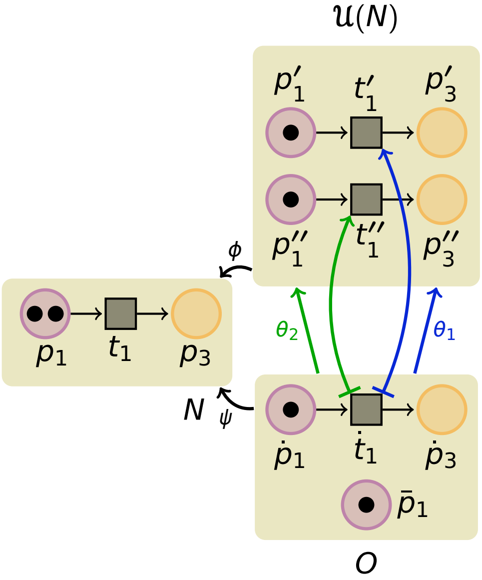

Solutions

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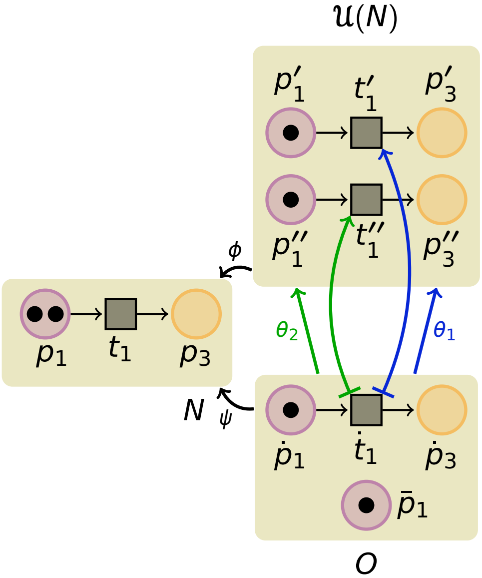

Solutions

- N is bad ⇝ safe / semi-weighted nets

NPW / MMS

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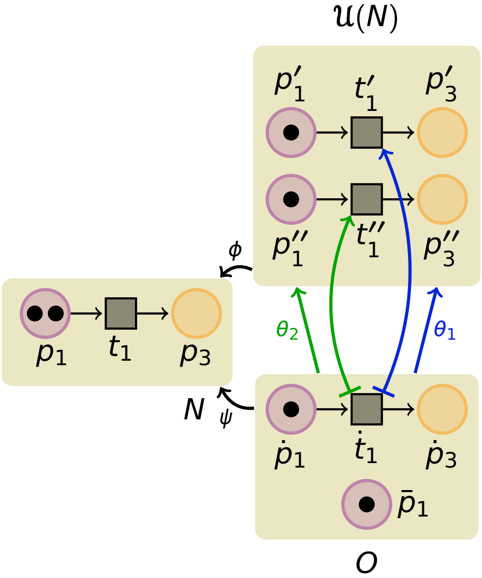

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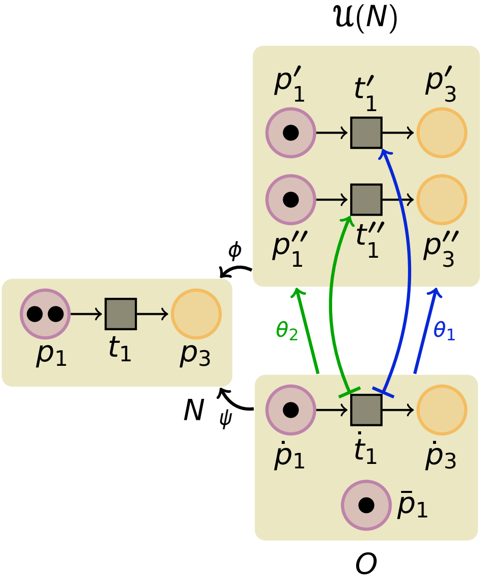

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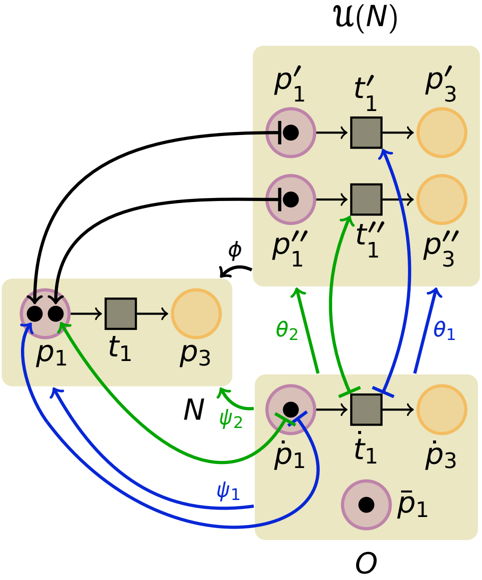

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- ϕ is bad ⇝ "disjoint" morphisms MMS
- θ_1 "=" θ_2 are (essentially) the same ⇝ bi-coreflection HW
- "individual" tokens B & morphisms with "extra information"

↳ application of our work

Overview

Motivation Unfolding as a Coreflector ✓

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From Petri Nets to Grammars

- pushout rewriting
- a category of grammars

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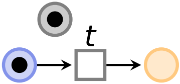
Conclusion

From Petri nets to pushout rewriting



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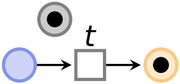

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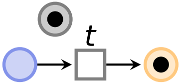
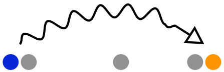


$${}^{\bullet}t \oplus m \;[t\rangle\; m \oplus t^{\bullet}$$



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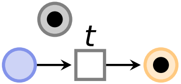

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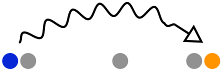
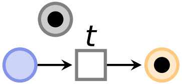


coloured sets $\mathbf{Set}{\downarrow}P$

$$L \uplus X \;\; \supseteq \;\; X \;\; \subseteq \;\; X \uplus R$$

From Petri nets to pushout rewriting



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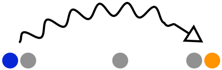

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$$\begin{array}{ccccc} L & & & & R \\ \downarrow & & & & \downarrow \\ L + X & \longleftarrow & X & \longrightarrow & X + R \end{array}$$

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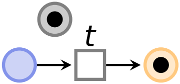

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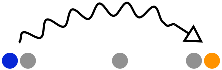
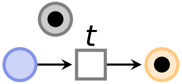


coloured sets $\mathbf{Set}\!\downarrow\! P$

$$
\begin{array}{ccccc}
\boldsymbol{L} & \longleftarrow & 0 & \longrightarrow & \boldsymbol{R} \\
\downarrow & & \downarrow & & \downarrow \\
\boldsymbol{L} + \boldsymbol{X} & \longleftarrow & \boldsymbol{X} & \longrightarrow & \boldsymbol{X} + \boldsymbol{R}
\end{array}
$$

From Petri nets to pushout rewriting



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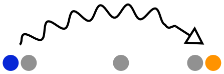

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$$\begin{array}{ccccc} L & \longleftarrow & 0 & \longrightarrow & R \\ \downarrow & & \downarrow & & \downarrow \\ L + X & \longleftarrow & X & \longrightarrow & X + R \end{array}$$

rule $q =$ $L \longleftarrow K \longrightarrow R$

match m

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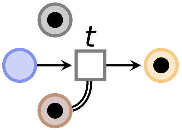

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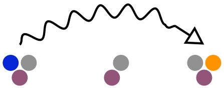

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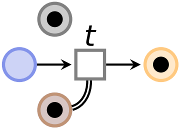

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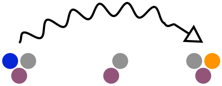

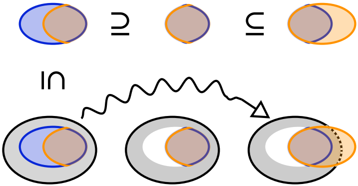

coloured sets $\mathbf{Set}{\downarrow}P$

"topological" objects

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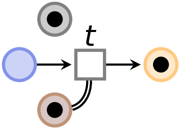

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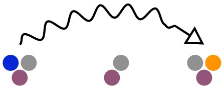

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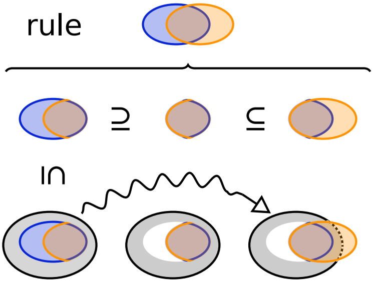

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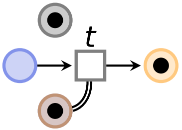

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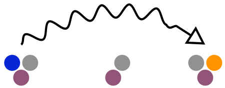

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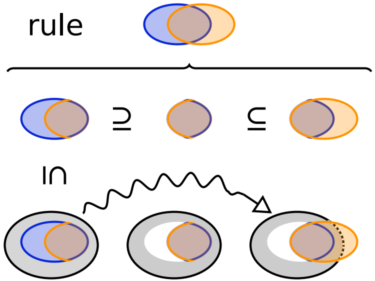

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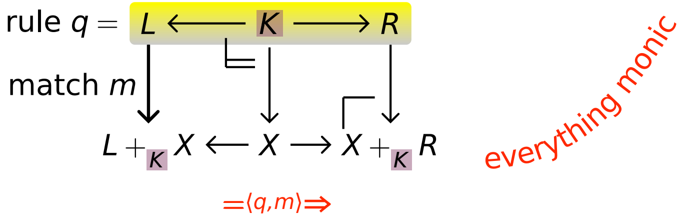
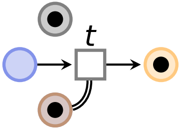
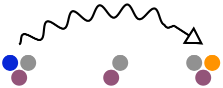


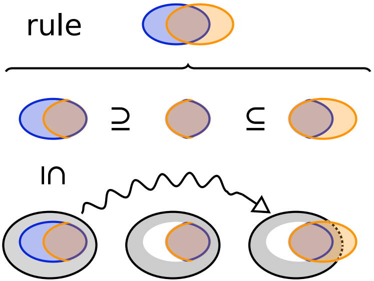
From Petri nets to pushout rewriting



$\underset{\sim}{t} \oplus {}^{\bullet}t \oplus m \ [t\rangle \ m \oplus t^{\bullet} \oplus \underset{\sim}{t}$



coloured sets **Set**$\downarrow P$



"topological" objects

$$\text{rule } q = L \longleftarrow K \longrightarrow R$$

match m

$$L +_K X \longleftarrow X \longrightarrow X +_K R$$

$=\langle q,m\rangle\Rightarrow$

adhesive categories

everything monic

Grammars: a generalization of Petri nets

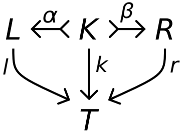
T

Definition (T-typed grammar ($T \in \mathbf{C}$))

Petri net $N = (P, T, {}^\bullet(\_), (\_)^\bullet)$
with
initial marking $m \in P^\oplus$

$P \triangleq T$

Grammars: a generalization of Petri nets



$\cdots$

Definition (T-typed grammar ($T \in \mathbf{C}$))

A *T-typed grammar* is a pair $G = (Q, s)$ with

1. Q a set of rules $l \leftarrow\!\alpha\!\prec k \succ\!\beta\!\rightarrow r$ in $\mathbf{C}{\downarrow}T$,

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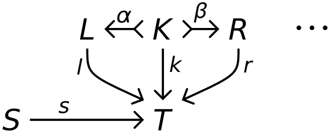
$\{({}^\bullet t, t^\bullet) \mid t \in T\} \triangleq Q$

Grammars: a generalization of Petri nets



Definition (T-typed grammar ($T \in \mathbf{C}$))

A *T-typed grammar* is a pair $G = (Q, s)$ with

1. Q a set of rules $l \leftarrow\alpha\prec k \succ\beta\rightarrow r$ in $\mathbf{C}{\downarrow}T$, and
2. $s\colon S \to T$ a morphism, i.e. $s \in \mathbf{C}{\downarrow}T$.

Petri net $N = (P, T, {}^\bullet(\_), (\_)^\bullet)$
with
initial marking $m \in P^\oplus$

$$P \triangleq T$$
$$\{({}^\bullet t, t^\bullet) \mid t \in T\} \triangleq Q$$
$$m \triangleq s$$

Unfolding à la Winskel

$$\mathbf{SafeNets} \underset{\supseteq}{\overset{\mathcal{U}}{\rightleftarrows}} \mathbf{OccNets} \qquad (\top)$$

Petri net morphisms are $\begin{cases} \text{multirelations } (\approx \textbf{Set}\text{-spans}). \\ \text{monoid homomorphisms.} \end{cases}$

Unfolding à la Winskel

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- multirelations (≈ **Set**-spans).
- monoid homomorphisms.

Grammar morphisms are
- **C**-span based,
- (monoidal) functors.

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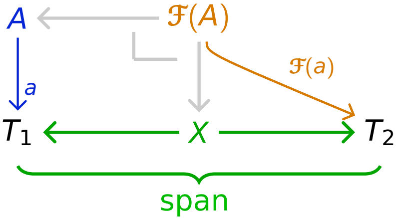
$\mathfrak{F}\colon \mathbf{C}{\downarrow}T_1 \to \mathbf{C}{\downarrow}T_2$

retyping functor



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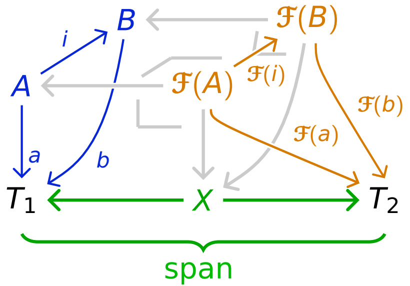

Category of grammars **Grm**

objects grammars $\langle Q, s\colon S \to T\rangle$ with $T \in \mathbf{C}$

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morphisms from $\langle Q_1, s_1\colon S_1 \to T_1\rangle$ to $\langle Q_2, s_2\colon S_2 \to T_2\rangle$:

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$$T_1 \longleftarrow X \longrightarrow T_2$$

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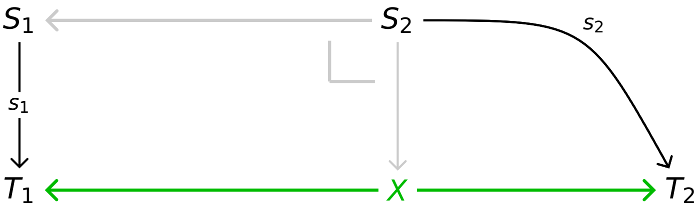

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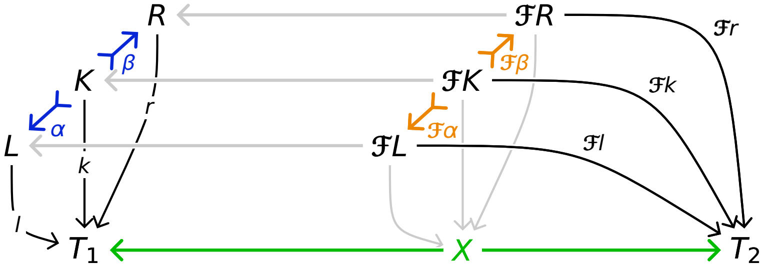
$$\mathfrak{F}(s_1) = s_2$$

and

$$\mathfrak{F}(q) \in Q_2$$

for all $q = (l \xleftarrow{\alpha} k \xrightarrow{\beta} r) \in Q_1$.



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$$\mathfrak{F}(q) \in Q_2 \cup \{\mathfrak{F}(k) \leftarrow \mathfrak{F}(k) \rightarrow \mathfrak{F}(k)\}$$

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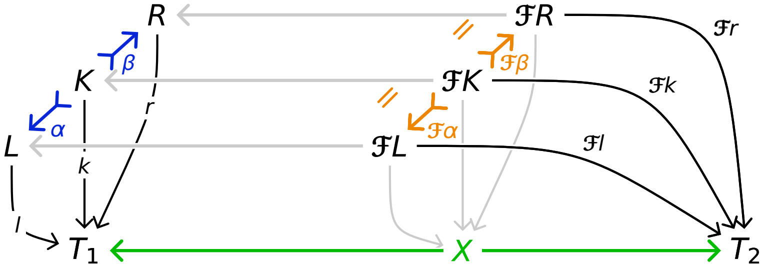
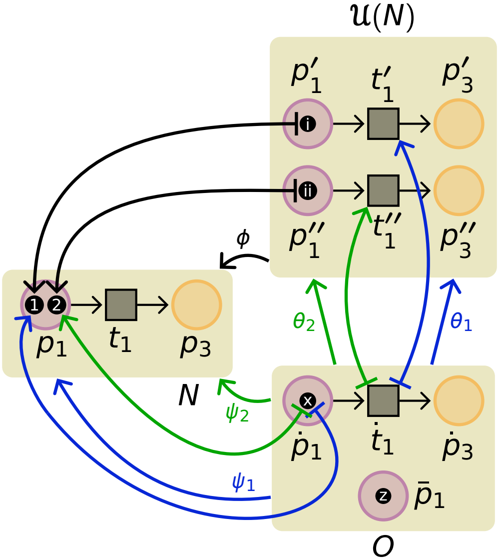
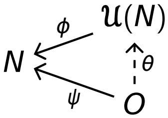
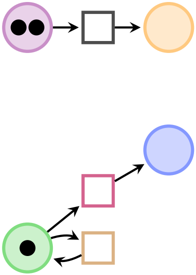


Coreflection in the example revisited





Unfolding . . .



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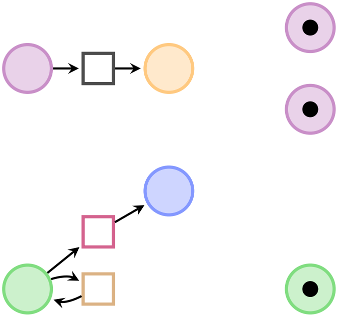

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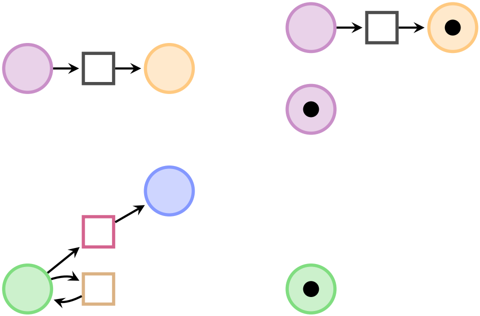

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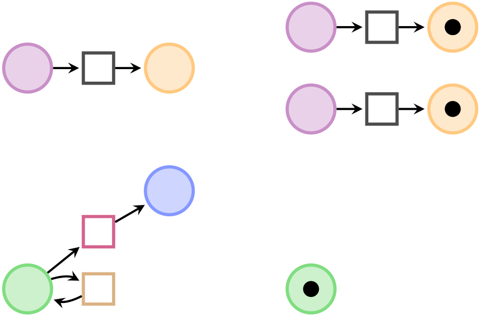

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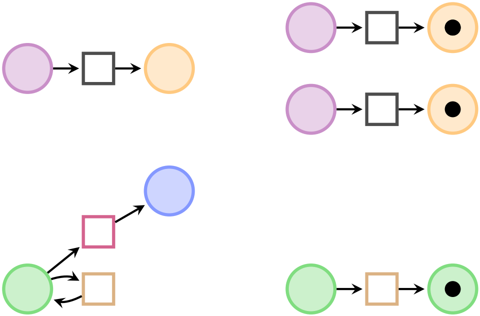

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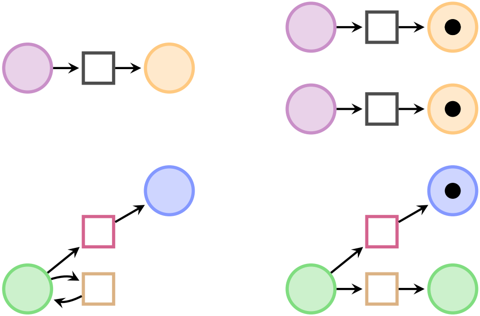

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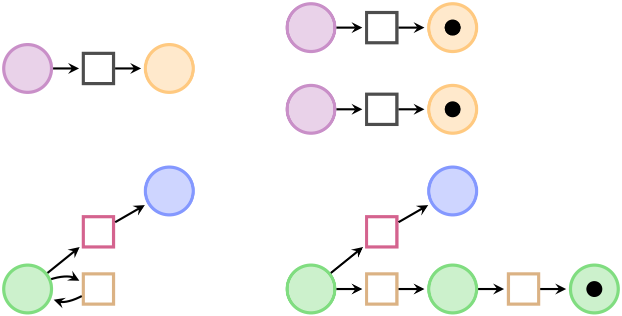

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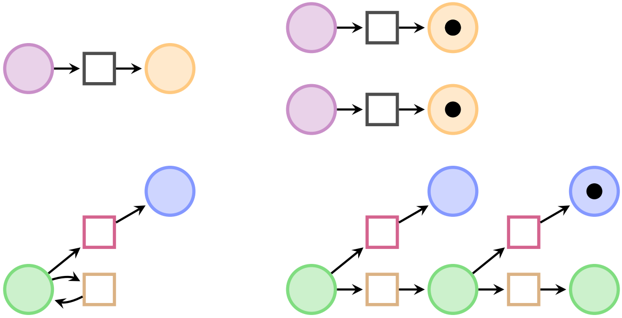

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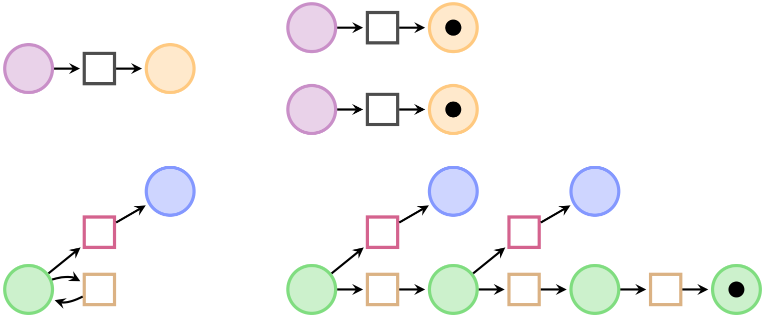

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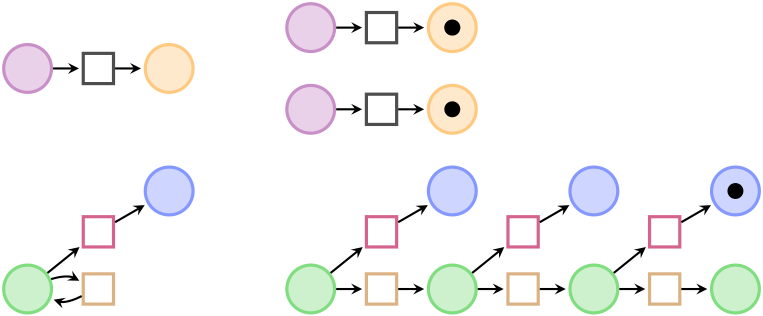

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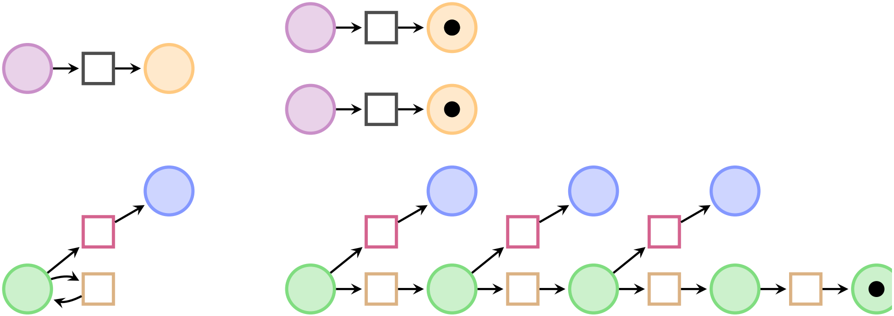

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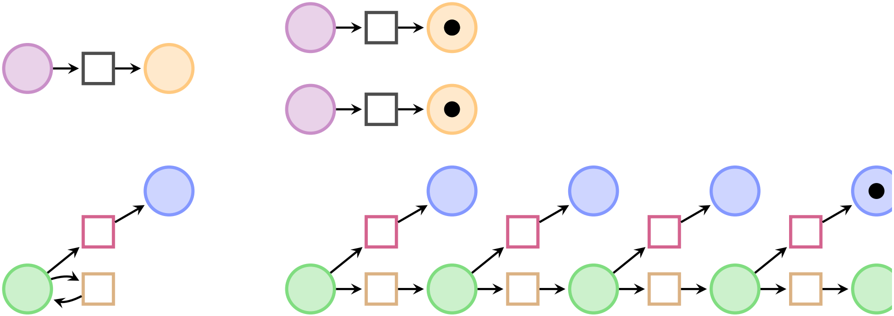

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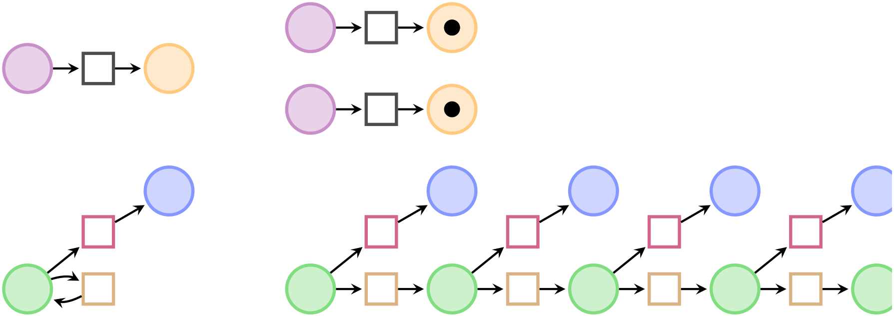

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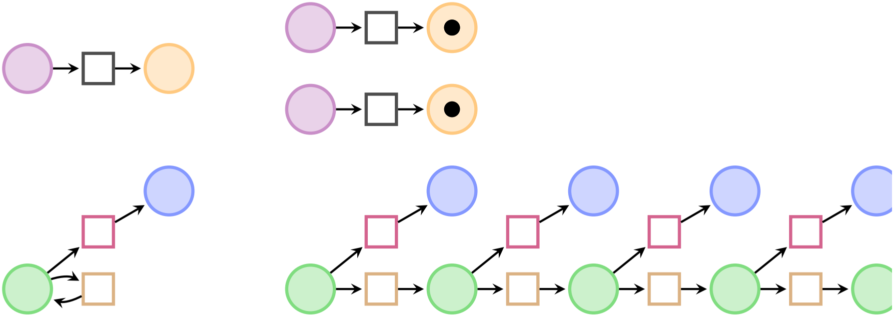


transition occurrences ⇝ rule occurrences

+ +

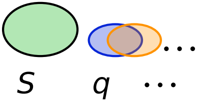
initial marking ⇝ start object

union + + ω-chain colimit

post-sets ⇝ right hand sides

Unfolding: “first” rule occurrences

grammar $\langle Q, s \colon S \to T \rangle$,
rule $q = (l \xleftarrow{\alpha} k \xrightarrow{\beta} r) \in Q$



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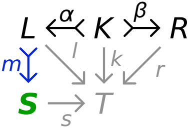
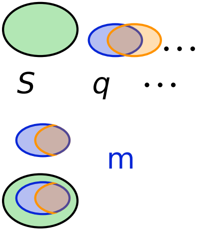


C$\downarrow T$ match m



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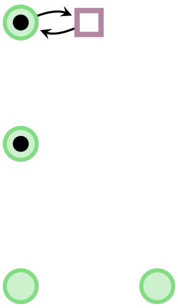

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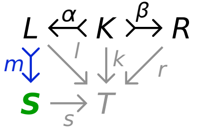
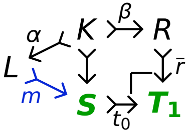
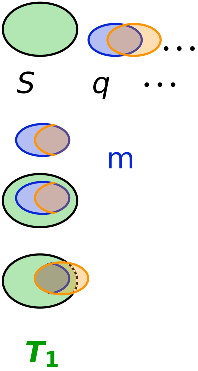


$\mathbf{C}{\downarrow}T$ match m



new
type object

$T_1 = S +_K R$



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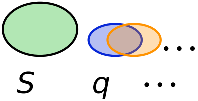

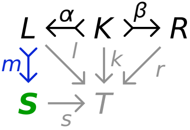

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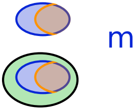
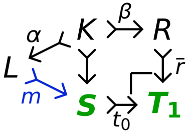
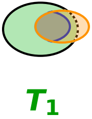
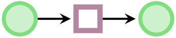
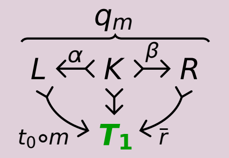




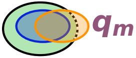
$T_1 =$ new type object $S +_K R$



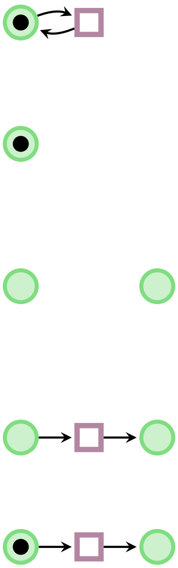




rule occurrence



Unfolding: “first” rule occurrences



grammar $\langle Q, s\colon S \to T\rangle$,
rule $q = (l \leftarrow\alpha\prec k \succ\beta\rightarrow r) \in Q$

$$\begin{array}{ccccc} L & \xleftarrow{\alpha} & K & \xrightarrow{\beta} & R \\ {\scriptstyle m}\downarrow & \searrow^{l} & \downarrow^{k} & \swarrow^{r} & \\ S & \xrightarrow{s} & T & & \end{array}$$

$\mathbf{C}{\downarrow}T$ match m

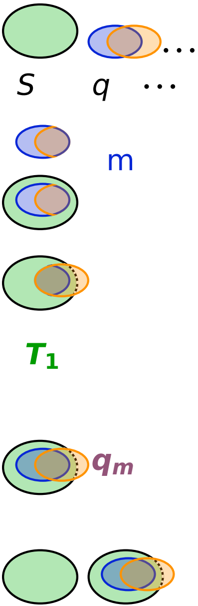
$$\begin{array}{ccccc} & & K & \xrightarrow{\beta} & R \\ L & \xleftarrow{\alpha} & \downarrow & & \downarrow \bar{r} \\ & \xrightarrow{m} & S & \xrightarrow{t_0} & T_1 \end{array} = S +_K R$$

new type object

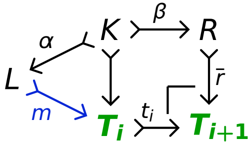
$$q_m = \overbrace{L \xleftarrow{\alpha} K \xrightarrow{\beta} R}, \quad t_0 \circ m\colon L \to T_1,\ K \to T_1,\ \bar{r}\colon R \to T_1$$

rule occurrence

$U_1 = \langle \{q_m\}, t_0\colon S \rightarrowtail T_1\rangle$

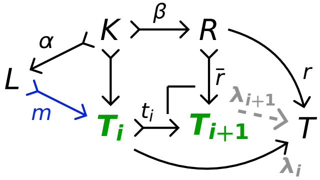


Unfolding: general unfolding step



$$U_i = \langle \{\ldots, \boldsymbol{q^i}\}, s_i \colon \boldsymbol{S} \rightarrowtail \boldsymbol{T_i} \rangle$$
$$\rightsquigarrow U_{i+1}$$

Unfolding: general unfolding step

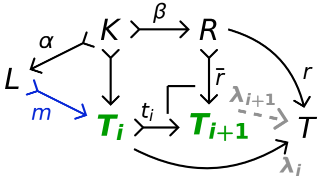


folding morphism $\boldsymbol{\lambda_i} \colon \boldsymbol{T_i} \to T$

$U_i = \langle \{ \ldots, \boldsymbol{q^i} \}, s_i \colon \boldsymbol{S} \rightarrowtail \boldsymbol{T_i} \rangle$

$\rightsquigarrow U_{i+1}$

Unfolding: general unfolding step



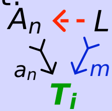
folding morphism $\lambda_i : T_i \to T$

$U_i = \langle \{ \dots, q^i \}, s_i : S \rightarrowtail T_i \rangle$

$\rightsquigarrow U_{i+1}$

match m: "coverable"/concurrent subobject!

$$s_i \overset{q_1^i}{\Longrightarrow} a_1 \cdots \overset{q_n^i}{\Longrightarrow} a_n$$



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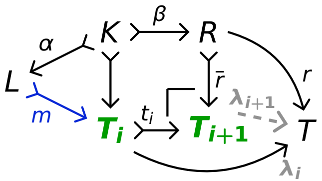


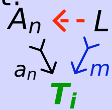
folding morphism $\lambda_i : T_i \to T$

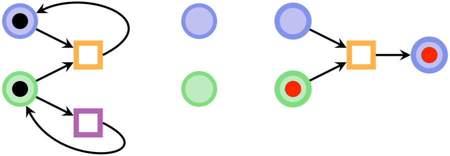
$U_i = \langle \{ \ldots, q^i \}, s_i : S \rightarrowtail T_i \rangle$

$\rightsquigarrow U_{i+1}$

match m: “coverable”/concurrent subobject!

$$s_i = q_1^i \Rightarrow a_1 \cdots = q_n^i \Rightarrow a_n$$





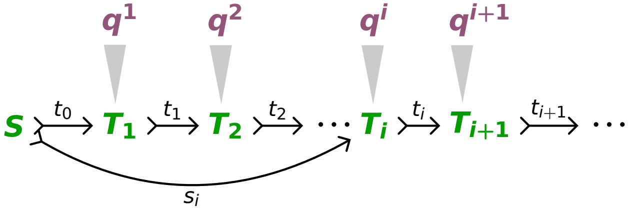
not coverable

⇒ no new occurrence

of □

Fair sequence of unfoldings

$$U_i = \left\langle \{\ldots, \boldsymbol{q^i}\},\ s_i \colon \boldsymbol{S} \rightarrowtail \boldsymbol{T_i} \right\rangle$$



The final construction step

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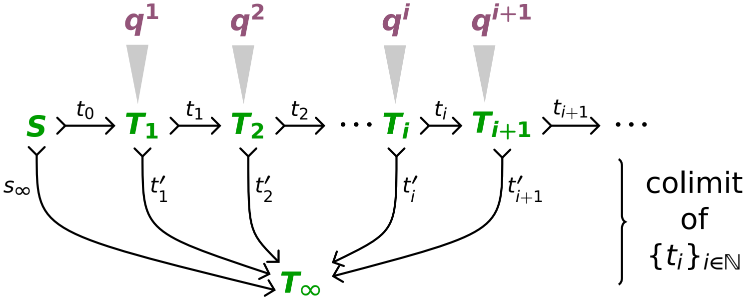

$$U = \left\langle \left\{ t_i' \circ \boldsymbol{q^i} \mid i \in \mathbb{N} \right\},\ s_\infty : \boldsymbol{S} \rightarrow \boldsymbol{T_\infty} \right\rangle$$

The final construction step

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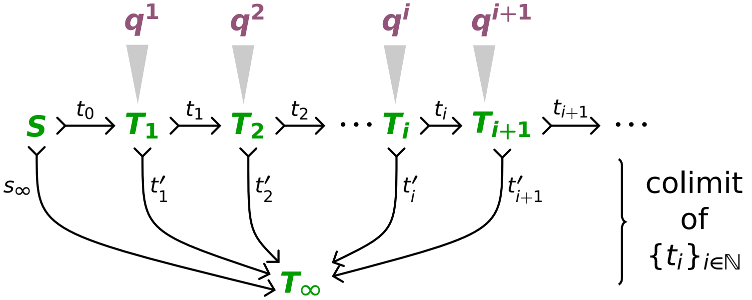

$$U = \left\langle \left\{ t'_i \circ q^i \mid i \in \mathbb{N} \right\},\ s_\infty : S \to T_\infty \right\rangle$$

. . . folding morphism $\lambda : T_\infty \to T$

The coreflection result

Unfolding produces

"safe" grammars with acyclic causality.

$\rightsquigarrow$ *occurrence grammars*

$$\mathbf{Grm} \xrightarrow{\mathcal{U}} \mathbf{OccGrm}$$

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Theorem

Unfolding of grammars is a coreflector.

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Theorem

Unfolding of grammars is a coreflector.

$\rightsquigarrow$ static notion of coverability

$\rightsquigarrow$ system composition:

$$\mathfrak{U}(G \times H) = \mathfrak{U}(G) \times \mathfrak{U}(H)$$

Conclusion and discussion

Conclusion

- *alternative approach to obtain coreflective unfolding semantics for PT nets*
- *grammar morphisms as span-based functors for unfolding as a coreflector*

(e.g. in any topos with countable sums)

$\rightsquigarrow$ Heindel (PhD) ✓

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Thanks !

Discussion

. . .